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THE CONSTRUCTION OF A SENSITIVE GALVANOMETER FOR SPECTRO-BOLOMETRIC PURPOSES.

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SINCE 1896 the galvanometers used for bolographic purposes at the Smithsonian Astrophysical Observatory have been more or less improved every year at the hands of the Observatory staff, with the encouragement and approval of the director, Secretary Langley, who has so kindly introduced this paper, which is a description of the results so far reached. For bolographic purposes reflecting galvanometers of low resistance and comparatively short time of swing are best suited. Solar-energy spectra are now being taken at the Smithsonian Observatory with a galvanometer of 1.6 ohms resistance, usually employed at 1½ seconds time of single swing of the needle; though for certain very delicate experiments the time of single swing is sometimes as high as 10 seconds. It may then be said that it is aimed to measure the least possible current with a galvanometer of 1.6 ohms resistance at 10 seconds single swing of the needle. It is well to make

[1] A paper read before Section B of the American Association for the Advancement of Science, December 30, 1902, after introductory remarks by S. P. Langley, Secretary of the Smithsonian Institution. It is here published with his permission.

here a distinction between computed sensibility and working sensitiveness, for it may happen that an instrument of less computed sensitiveness is capable of measuring smaller currents than another whose high figure of sensibility is overbalanced by unsteadiness of its needle system. This consideration leads me to suggest that possibly it might be fairer to compare galvanometers on a basis of the least current which can be read with them, or at least to give this in addition to the usual statement of computed sensibility.

In common with other laboratories, the Smithsonian Observatory has greatly profited by the introduction of Professor Boys' quartz fibers and by diminishing the size of the needle system and mirror of the galvanometer. These improvements were introduced about 1893 in the galvanometer described by Mr. Wadsworth[1] and used for bolographic purposes up to the autumn of 1896.[2]

In September 1896 the Observatory was visited by Professor Kayser, of Bonn, who expressed the belief that a galvanometer of considerably greater sensitiveness might be employed. He referred especially to the excellent instrument of Paschen, and to a galvanometer which he had seen at Baltimore, of somewhat similar design to Paschen's. Acting upon this suggestion, Mr. Langley addressed a letter of inquiry to Professor Paschen, and he was so good as to send in reply quite an account of his instrument. I also went to Baltimore and saw the galvanometer being set up by Messrs. Mendenhall and Saunders. These instruments seemed to gain in sensitiveness chiefly by diminishing the dimensions of the needle system, and as an aid to this the dimensions of the coils were also small.

Beginning with the autumn of 1896 experimental and theoretical work has gone on in the Smithsonian Observatory at frequent intervals, with the object of improvement in all the

[1] *Phil. Mag.*, **38**, 553–558, 1894.

[2] As last employed with a needle system of 31 milligrams weight, constructed by the writer, this instrument of about 20 ohms resistance gave a deflection of 1 millimeter on a scale at 1 meter with a current of 6×10^{-10} ampères when the time of single swing was 4 seconds. At this time of swing the needle was ideally steady, so that deflections of 0.1 millimeter could readily be observed.

branches of galvanometer construction, including the coils, the needle system, the case, and the accessories. In what follows the principal results of this study will be stated.

THE COILS.

Best resistance.—The bolometric circuit with which the galvanometer is connected, consists, as is well known, of a balanced Wheatstone's bridge. In our practice one of the four resistances of this bridge is a fine flattened platinum wire subjected to changes of temperature through the absorption of radiation. For the sake of symmetry a second one of the four resistances is a similar strip of platinum close to the first, but protected from radiation by means of diaphragms. The other two resistances are coils of platinoid wire. The battery is connected between the junction of the two coils and the junction of the two strips, while the galvanometer is connected between the junction of one strip with one coil and the other corresponding point. It is shown in the *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vol. I, p. 246, that the condition of maximum galvanometer deflection requires the resistance of the two coils to be large in comparison with the strips.

Let a and $\left(a+\frac{a}{m}\right)$ be the resistances of the two bolometer strips; (m being a large number); na the resistance of each balancing coil; G the resistance of the galvanometer; C the current in the bolometer strip whose resistance is a; g the current flowing through the galvanometer; Δ the deflection of the galvanometer, and k a constant.

Then

$$g=\frac{C}{m}\frac{1}{2+\frac{G}{a}\left(\frac{1+n}{n}\right)}\,. \tag{1}$$

When, as with the coils shortly to be described,

$$\Delta=kgG^{0.45}\,, \tag{2}$$

we have by substitution

$$\Delta=k\frac{C}{m}\frac{G^{0.45}}{2+\frac{G}{a}\left(\frac{1+n}{n}\right)}\,. \tag{3}$$

In this expression Δ is a maximum with respect to n when n is infinitely great, and with respect to G when

$$\left(\frac{1+n}{n}\right)\frac{G}{a} = 1.64 \; . \qquad (4)$$

If n is large, this becomes approximately

$$\frac{G}{a} = 1.64 \; . \qquad (5)$$

The two following tables indicate how much is lost by departure from these conditions of maximum deflection:

n	¼	1	2	3	4	∞
Δ	44	71	82	87	89	100

$\left(\frac{1+n}{n}\right)\frac{G}{a}$	0.0164	0.164	0.82	1.64	3.28	16.4	164
Δ	23	60	95	100	94	56	18

It appears that the conditions of maximum sensitiveness of the bolometric circuit as used here are closely approximated when the balancing coils are upward of four times the resistance of the bolometer strips, and the galvanometer resistance is not less than six-tenths, or more than four times the resistance of the bolometer strip.

At the Smithsonian Observatory best results have been obtained with bolometers of comparatively low resistance, for, unless in an air-tight case, very thin strips are much affected by air-currents, so that a perpetual tremor of the galvanometer is observed when there is moderate wind. At present a bolometer with strips of 4 ohms resistance is chiefly employed, but occasionally a wide bolometer of only 0.8 ohms resistance is substituted. Accordingly a galvanometer of from 2 to 3 ohms resistance would be most serviceable, but the actual resistance of the galvanometers now in use is 1.6 ohms.

A four-coil galvanometer of a given total resistance G, may have its separate coils of resistance $4G$, G, or $\frac{G}{4}$, according to

the manner in which they are connected; and for coils whose force at the center varies with the 0.45 power of the resistance, the relative efficiency of these three ways of producing the total resistance G is in the ratio of the numbers 87, 93, and 100. Thus it appears that the resistance of a four-coil galvanometer may be varied in three steps from 1 to 16 without greatly altering its efficiency. Again a galvanometer of a given resistance may be made by connecting in series two, four, eight, sixteen, or even more such coils. The efficiency of these several arrangements of the given resistance, so far as the force at the center of the coils is concerned, is in the ratio of the numbers 100, 68, 49, 32. There is a slight compensation for this diminution of efficiency as the coils get smaller, arising from the greater effect of the outer windings of each coil upon the magnets at the center of its neighbor, but the only considerations which warrant a multiplication of coils are some which concern the efficiency of the magnet system.

Best form of coils.—Maxwell[1] has shown that the contour of the cross-section of the coil should be of the form determined by the equation

$$r^2 = H^2 \sin\theta \ , \tag{6}$$

where r is the length of a radius making the angle θ with the axis of the coil, and H the value of r when $\theta = 90°$. Although the efficiency of the winding is not very greatly impaired by considerable variations from the best form of coil section, it is so easy to follow it closely that this form has always been employed here. In order to leave room for the needle system, a space is left unwound at the center and front of each coil as indicated by dotted lines in the accompanying diagram (Fig. 1).

Best sizes of wire.—The reader is referred to *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vol. I, p. 248, for the derivation of equations required to determine the radius of a coil of the above form containing a given length of wire of a given diameter and gain of diameter by insulation, and the force such a coil exerts at its center. Maxwell has shown[2]

[1] MAXWELL, *Electricity and Magnetism*, Vol. II, paragraph 718.

[2] *Ibid.*, paragraph 719.

that the wire should increase in diameter from the center outward, and it has been customary therefore to wind the coils in several sections, with larger sizes toward the outside of the coil. Although there would be a slight gain by using a larger number, it has been the practice here to wind the coils in three sections as shown in the accompanying diagram (Fig. 1), which is drawn to scale to illustrate the best dimensions for a three-section coil of 5 ohms resistance. The following table is for the most part abridged from Table 32 of the Astrophysical Observatory *Annals*, Vol. I, and shows the diameters of wire, gain of diameter by single white silk insulation,[1] lengths of wire, external radii of sections and forces exerted at the center by the most efficient coils of various given resistances, when wound of wire of a single size or in three sections of wire of different diameters. The coils of maximum efficiency have been selected from a large number whose constants have been computed.

Total Resistance	Diameter of Wire in Sections			Gain of Diameter by Insulation	Length of Wire in Sections			External Radii of Sections			Forces at Center of Coil			Total Force at Center of Coil
	d_1	d_2	d_3	t	L_1	L_2	L_3	H_1	H_2	H_3	F_1	F_2	F_3	F
ohms	cm	cm	cm	cm	cm	cm	cm	cm	cm	cm				
0.1	0.0405	......	......	0.0038	79.1	.....	.....	0.55	....	....	....	....	....	104
"	0.0202	0.0405	0.0511	"	6.6	26.4	41.6	0.36	0.42	0.60	54	45	42	141
5	0.0101	0.0202	0.0405	"	81	328	1318	0.32	0.60	1.30	298	321	301	920
"	0.0101	0.0255	0.0644	"	81	521	3332	0.32	0.75	2.26	298	359	290	947
25	0.0160	......	......	"	3092	.....	.....	1.00	....	....	....	....	....	1467
"	0.0080	0.0160	0.0321	"	256	1031	4144	0.38	0.74	1.63	671	670	605	1946
"	0.0101	0.0202	0.0405	"	407	1639	6589	0.46	0.95	2.16	754	653	564	1971
50	0.0160	......	......	"	6185	.....	.....	1.24	....	....	....	....	....	1907
"	0.0080	0.0160	0.0321	"	513	2062	8288	0.45	0.91	2.03	991	913	784	2688
"	0.0101	0.0202	0.0405	"	815	3278	13180	0.55	1.17	2.70	1095	898	717	2710
250	0.0101	......	......	"	12225	.....	.....	1.23	....	....	....	....	....	4007
"	0.0050	0.0101	0.0202	"	1017	4075	16390	0.46	0.91	1.96	1871	1798	1645	5314
"	0.0050	0.0127	0.0321	"	1017	6483	41440	0.46	1.14	3.38	1871	2017	1617	5505
500	0.0101	......	.....	"	24450	....	.	1.53	....	....	....	....	....	5398
"	0.0050	0.0101	0.0202	"	2033	8150	32780	0.555	1.12	2.45	2731	2396	2126	7253
"	0.0050	0.0127	0.0321	"	2033	12970	82880	0.555	1.42	4.23	2731	2677	2068	7476

Taking the best coils given in the table, the total force exerted at the center is closely proportional to the 0.45 power

[1] Wire of the diameters given (which are those of standard sizes of the B & S wire gauge) can be procured with white silk insulation of the given thickness from the firm of A. F. Moore & Co., of Philadelphia.

of the total resistance, Coils composed of three sections of best sizes of wire give about 1.4 times the force at center given by a coil of the best single size of wire of the same total resistance.

Method of winding.—The method I have employed in winding such coils is as follows: A brass mandrel is prepared as shown in the diagram (Fig. 1). Where coils of large resistance are to be wound the ends of wires of the several sections are brought out at the back of the coil and soldered after its completion, but for 0.1-ohm coils it is better to solder the wires together before winding. The wire is wound in a solution of shellac, and the sections are formed correctly and held in place till the shellac dries by winding thread against the back of the section as it grows in radius. After the coil is complete it is boiled in carnauba wax to make it rigid, after which the front plate of the mandrel is warmed and removed, and then the mandrel itself is warmed while the coil is held downward in cold water till it can be drawn from the mandrel.

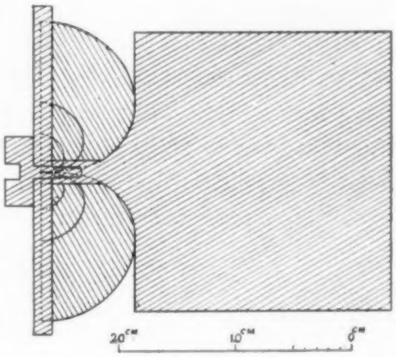


FIG. 1.

THE NEEDLE SYSTEM.

The great sensitiveness of recent galvanometers of the Thomson type in comparison with those of twenty years ago is much more due to the small dimensions of the needle system, than to improvements in the coil construction. Paschen[1] has emphasized this point, and he and others had constructed galvanometers of great sensitiveness prior to the time when the present investigations began. More recently Mendenhall and Waidner,[2] in a valuable paper on galvanometer construction, have described more thorough studies of the construction of needle systems, and point out that the useful diminution of dimensions cannot proceed as far as would appear from Paschen's paper.

[1] *Zeitschrift für Instrumentenkunde*, **13**, 13, 1893.

[2] *Am. Jour. Sci.* (4), **12**, 249, 1901.

In their construction of magnet systems these authors are guided by extensive experiments on magnets of different forms, but they have not given exact directions for the dimensions and arrangement of the whole needle system to give highest sensitiveness. In what follows I shall endeavor to furnish exact information of this kind.

If we suppose a single short magnet suspended horizontally at the center of a coil to be influenced in a direction parallel to the plane of the coil by a magnetic field of strength H, and to be influenced at right angles to the plane of the coil by a current field of strength F, it will take up a direction making an angle θ with the plane of the coil such that

$$\tan \theta = \frac{F}{H} . \tag{7}$$

Let T be the time of swing of the suspension (undamped), M its magnetic moment, and I its moment of inertia, and let F be small as compared with H, then

$$T^2 \propto \frac{I}{MH} . \tag{8}$$

Combining (7) and (8)

$$\tan \theta \propto \frac{FMT^2}{I} . \tag{9}$$

For a given current and time of swing therefore

$$\tan \theta \propto \frac{M}{I} . \tag{10}$$

It is usual for purposes of astaticism to employ more than one group of magnets, with poles in the several groups oppositely directed. Suppose n such groups each composed of p magnets of moment M and let ΣMp be their algebraically combined magnetic moment tending to direct the system in a given sense, when $F = O$, while nMp is their total magnetic moment without regard to sign. Let each group hang at the center of a coil whose field strength is f, and let H, T, and I have the same significance as before.

Then

$$\tan \theta = \frac{npfM}{H \Sigma Mp} . \tag{11}$$

If, as before, $npfm$ is small as compared with $H\Sigma Mp$

$$T^2 \propto \frac{I}{H\Sigma Mp} . \qquad (12)$$

Whence for a given current and time of swing

$$\tan\theta \propto \frac{npM}{I} . \qquad (13)$$

The moment of inertia I falls naturally in two portions, viz., the non-magnetic part which may be called I_n, and the magnetic part which is equal to npI_m where I_m is the moment of inertia of each separate magnet. Let $np = N$ the total number of magnets.

Thus the expression (13) becomes

$$\tan\theta \propto \frac{NM}{NI_m + I_n} . \qquad (14)$$

Of I_n all that need be said at present is that obviously it should be as small as practicable, and may be regarded as a known constant. If then M and I_m can be expressed in common terms the conditions of maximum deflection can be determined. It is easy to express I_m in terms of the linear dimensions of the magnets; thus for a magnet of length $2a$, weight w, and radius r cemented to a thin vertical glass stem, the moment of inertia is approximately $\frac{wa^2}{3}$ when $\frac{a}{r}$ is not less than 7.

Magnetic moment in terms of linear dimensions.—In order to find an expression for the magnetic moment in terms of the linear dimensions of the magnet, many experiments have been conducted here, which have led to a useful expression of this kind. As the method of measuring relative magnetic moments is useful in testing magnets used in needle systems, I shall briefly describe it. A single short magnet provided with a mirror is suspended in a field controlled in such a way that the time of swing of the suspension is raised to one or two seconds. The magnets to be tested are then brought up from the rear, end on, at right angles to the suspended magnet, at the same height with it and to a fixed distance which should be large compared with the length

of the magnets used. In this case the deflections of the suspension are directly proportional to the magnetic moments of the magnets approached.

In the experiments the relative magnetic moments of numerous magnets of different kinds, forms, and sizes were thus determined. The magnets varied in weight between 368 mg and 0.06 mg; in length between 22 mm and 0.8 mm; in diameter between 1.7 mm and 0.075 mm. Some were flat, some short and thick, others as much as fifty times as long as their diameters. Not only were they tried separately, but in combinations of from two to twenty, separated by spaces of from zero to five diameters, sometimes magnetized separately and then approached, at other times magnetized together.

The following table illustrates the advantage of dividing a magnet of a given weight into a number of thinner ones. The magnets were all from the same bar of steel tempered at once and magnetized to saturation:

TABLE I.

Designation of Magnet	Length mm	Width mm	Thickness mm	Weight	Magnetic Moment (Arbitrary Unit)
A_1	6.1	3.0	0.35	0.0425	7.3
A_2	6.1	0.88	0.35	......	4.9
A_3	6.1	0.88	0.35	Total weight	
A_4	6.1	0.88	0.35	of four	
A_5	6.1	0.88	0.35	0.0507	
A_2 and A_3 placed parallel and a little apart........................					9.3
A_2 A_3 and A_4 do. do..					13.4
A_2 A_3A_4 and A5 do. do...............................					18.0

We see that the four magnets had a combined magnetic moment two and one-half times as great as the single one of five-sixths their combined weight. Other experiments showed that a given weight of steel continued to increase in magnetic moment with increasing ratio of length to mean diameter even when the ratio was 50 to 1. A round or square cross-section appears preferable to a flat one.

Thoroughly to determine the connection between magnetic

moment and dimensions, a certain piece of steel was drawn into wire of four different sizes. Pieces were taken from each wire and all hardened at the same time. These were then magnetized to saturation together in a long helix. After determining the magnetic moment of each they were broken up into short lengths and again magnetized as before and the magnetic moment of each piece determined separately. The results are given in the following table:

TABLE II.

Designation	Length $2a$ cm	Diameter $2r$ cm	Weight W gr	Magnetic Moment M (Arbitrary Unit)	Ratio $\frac{2a}{2r}$	Ratio $10^5 \frac{W}{M} \frac{2a}{2r}$	Deviations from Mean
A_1	0.80	0.0266	0.00297	130.0	30.1	68.7	−29.2
A_2	0.30	0.0266	0.00111	9.7	11.3	117.4	+19.5
A_3	0.13	0.0266	0.00048	1.8	4.9	130.0	+32.1
B_1	0.73	0.0181	0.00138	59.0	40.3	93.5	− 4.4
B_2	0.35	0.0181	0.00066	19.0	19.4	67.0	−30.9
B_3	0.13	0.0181	0.00025	1.3	7.2	138.0	+40.1
B_4	0.08	0.0181	0.00015	0.7	4.4	94.6	− 3.3
C_1	0.33	0.0137	0.00037	8.4	24.1	94.4	− 3.5
C_2	0.08	0.0137	0.00009	0.5	5.8	96.9	− 1.0
D_1	0.50	0.0075	0.00015	9.4	66.6	106.4	+ 8.5
D_2	0.19	0.0075	0.00006	2.1	25.4	70.0	−27.9
					Mean	97.9	18.1

It will be seen that in this series of experiments the weight of the magnets varied fifty fold, the ratio of length to diameter varied from 66.6 to 4.4, and the magnetic moment varied 250 fold. In all this range of values the magnetic moment was proportional to the product of the weight by the ratio of length to diameter, within an average deviation of about 18 per cent. While this average deviation is large, it does not indicate a departure from the numerical relation just pointed out, for there appears on the whole no tendency for the deviations to become positive or negative either for the heavier or lighter needles or for those longest or shortest in proportion to their diameter. The large deviations may reasonably be explained as caused by differences in the magnetic quality of the steel, as will appear in a later page. Numerous experiments not here given have con-

firmed this general result, and in some cases series have been considerably more accordant than the one here given. It will be interesting to see whether the results of Mendenhall and Waidner support this relation. The following table is roughly computed from the series of magnets H and A given by them.[1]

TABLE III.

Designation	Length $2a$	Mean Diameter[2] $2r$	Weight W	Magnetic Moment M	Ratio $\frac{2a}{2r}$	Ratio $100\,\frac{W}{M}\,\frac{2a}{2r}$	Deviations from Mean
	cm	cm	g				
H_1	0.530	0.010	0.000430	0.0258	53.0	88.5	+23.6
H_2	0.415	0.010	0.000337	0.0184	41.0	75.2	+10.3
H_3	0.316	0.010	0.000251	0.0122	32.0	65.8	+ 0.9
H_4	0.195	0.010	0.000155	0.0048	19.0	61.3	− 3.6
H_5	0.124	0.010	0.000100	0.00215	12.0	55.9	− 9.0
H_6	0.102	0.010	0.000081	0.00143	10.0	56.5	− 8.4
H_7	0.065	0.010	0.000053	0.00054	6.5	62.9	− 2.0
					Mean	64.9	8.3
A_1	0.335	0.024	0.00157	0.0407	13.0	50.0	+ 2.2
A_2	0.229	0.024	0.00105	0.0198	8.8	46.9	− 0.9
A_3	0.167	0.024	0.00078	0.0100	6.4	50.0	+ 2.2
A_4	0.080	0.024	0.00036	0.0031	3.2	37.7	−10.1
A_5	0.153	0.017	0.00034	0.0049	9.0	59.9	+12.1
					Mean	47.8	5.5

These results show closer agreement to the relation above given than does the series of observations before quoted. There appears, however, a tendency to departure from the formula for magnets more than forty and for those less than five times as long as their diameter. I shall therefore restrict my statement as follows: For magnets of a given steel tempered and magnetized to saturation under like conditions, and whose length is between five and forty times their diameter, the magnetic moment is proportional to the product of the weight by the ratio of length to diameter.

Mutual action of magnets.— My observations have led me to a

[1] *Am. Jour. Sci.*, **12**, 256, 1901.

[2] The square root of the product of width and thickness.

different view of the seriousness of mutual action between magnets in needle systems from that arrived at by Mendenhall and Waidner, who incline to think it of little consequence. In the table of experimental results which they give the distance between centers of the magnets was never less than 1.0 mm and the magnets were flat and were 0.8 mm wide. Had they been turned up edgewise and approached closer, I think the mutual effect would have greatly increased. The following experiments were made with magnets of round wire about 1.4 mm long and 0.08 mm in diameter, being of the same dimensions that I am accustomed to use for needle systems. Ten of these were magnetized to saturation in a long helix, and gave (in arbitrary units) magnetic moments of 15, 13, 15, 12, 14, 13, 14, 13, 11, and 17, respectively, when measured separately, so that the arithmetical sum of their moments was 137 units. When placed in a bunch parallel and touching each other, the combined moment was only 35 units. The magnets were then again tested separately and had permanently decreased in magnetic moment to only five units each. They were then built into a system with spaces of three diameters between them, and were re-magnetized in the helix. This system gave a combined moment of 105 units. At two diameters and one diameter separation the moment became 95 and 74 units, respectively. Further experiments showed that similar results were reached whether the magnets were first approached and then magnetized, or *vice versa*. With spaces of five diameters between the magnets the loss of combined moment by mutual effect was only about 3 per cent.

Comparative magnetic moment of different samples of steel.— Different kinds of steel are unequally efficient for magnet systems. The best kind tried here was a small bar of tungsten steel of a composition unknown to me, which was given to me at Johns Hopkins University in Baltimore. As compared with Stubs' tool steel, this made up into magnets of twice the magnetic moment for equal weight and similar form. I have used it exclusively in recent galvanometers. But it is not alone samples of steel of different composition, but different samples

from the same bar, that show unequal magnetic value. Thus among 140 magnets of tungsten steel wire 0.008 cm in diameter and all between 0.125 and 0.135 mm long, broken from the same wire, tempered at the same time, and magnetized in a long helix at once, the magnetic moment was found to vary between the limits 13 and 35. Sixty-four, however, were selected whose average magnetic moment was 30 on this arbitrary scale, and most of the remainder were between the limits 20 and 25. It is undoubtedly to this variation of small samples from the same bar that the large deviations in Table II are due.

The following is a summary of the general results reached in these experiments on magnetic moment.

1. Rowland's tungsten steel made the strongest magnets of all the kinds tried, and proved twice as good as Stubs' tool steel.

2. A given weight of steel used to compose a single magnet has a greater magnetic moment if square or round in cross-section than if flat, and the magnetic moment is greater the greater the ratio of the length to the diameter.

3. A given weight of steel of a given length is increased in magnetic moment the more it is subdivided lengthwise to form separated magnets.

4. For magnets of the same steel between five and forty times as long as their diameter, the magnetic moment is given by the relation $M = K' \frac{Wa}{r}$, where M is the magnetic moment, W the weight, a the half length, r the radius, and K' a constant for the given kind of steel. This important relation was found to hold throughout the great range of weights and forms of magnets examined.

5. A number of magnets placed parallel in a group reduce each other's magnetic moments by about one-thirtieth if separated by spaces of 5 diameters, and the loss becomes about one-third at two diameters. It is undesirable to have the spaces less than three diameters.

6. It is immaterial whether the magnets are first magnetized and then approached, or first approached and then magnetized, for the loss is not recovered when the magnets are separated.

We now are in position to determine the best construction of needle systems; for, remembering that the magnetic moment is proportional to $\frac{NWa}{r}$, the moment of inertia to

$$\frac{NWa^2}{3} + I_N ,$$

and the deflection of the galvanometer to their quotient we have

$$\tan\theta \propto \frac{NW\frac{a}{r}}{\frac{NWa^2}{3} + I_N} . \tag{15}$$

I have used this formula to compute numerical values for a series of systems, selecting for construction one which is as heavy as possible without sacrificing more than one-tenth in sensitiveness as compared with the most sensitive. In this way better results are reached than by substituting for W in terms of the linear dimensions and specific gravity, and determining analytically the conditions of maximum sensitiveness.

We see by the formula that if the total weight of the system, the length of the needles, and the non-magnetic moment of inertia are all kept constant, the deflection increases indefinitely by diminishing the diameter of the magnets. But a practical limit is soon reached in this course of procedure, for in order to remain effective, the magnets must be separated by three or four diameters, and thus the space available for them is soon taken up. In practice I now employ magnets 0.008 centimeters in diameter.

By inspection of the formula it is also apparent that if the length and radius of the magnets and the non-magnetic moment of inertia all remain unchanged, an increase in the number of magnets increases the deflection. The great advantage of increasing the number is, however, that the non-magnetic moment of inertia can also be increased at the same time, and thus both the weight and rigidity of the needle system is augmented without loss of sensitiveness, so far as the needle system is concerned, and with a real gain in steadiness. But the limitation of space at the center of the coils bars indefinite progress in this

direction. In the two galvanometers now in use at the Observatory this increase in number of magnets has been effected by increasing the number of groups of magnets from two to eight, with an attendant increase of the number of coils from four to sixteen.[1] As shown at a former page, this increase in the number of coils diminishes the effectiveness of the current in the ratio of 68 to 32.

The "constant" of one of these sixteen-coil instruments expressed in ampères for a deflection of 1 mm on scale at 1 meter with time of single swing of 10 seconds and total resistance only 1.6 ohms is 5×10^{-11}.[2]

It is well to remark that, while the sixteen-coil instrument involves a deliberate sacrifice in the efficiency of the coils, it gains greatly in its better astaticism and the steadiness of the needle system. The least current which can be read with it as now arranged is about 1×10^{-12} ampères.

For a region free from prejudicial magnetic disturbances it would be better to retain the four-coil form of galvanometer. By making the coils rather large the interior unwound space could be a centimeter or more in diameter without great sacrifice of current effectiveness, and a large number of needles could be placed in each group.

Building of needle systems.—The method of building up a needle system of fifty or more magnets deserves a little explanation. In the first place the steel wire is cut off in three-quarter-inch lengths and hardened glass hard between thin

[1] A needle system of 10 mg weight having 48 needles each 0.132 cm long and 0.0082 cm in diameter, fixed to one side only of a stem 12 cm long and weighing complete with magnets and mirror 8 mg, is now in use. The several moments of inertia of its materials are: of magnets, 0.0000045 gr cm; of mirror, 0.0000005; of stem, 0.0000012; of shellac, 0.0000002. The practice of putting needles on both sides of the stem far enough apart to avoid loss of magnetic moment by mutual action increases the moment of inertia of the system too much to be of advantage.

[2] One needle system of 2 mg weight for a four-coil instrument of 1.4 ohms at 1.5 seconds single swing gave a millimeter deflection on the scale at one meter distance, with a current of 4×10^{-10} ampères. If we compute from this to a ten-second single swing the result is 9×10^{-12}, and if to a ten-second complete period the result is 24000 on the Ayrton-Mather scale. But both these computed values are illusory, for with atmospheric pressure the deflection of light systems is not even approximately proportional to the square of the time of swing. As shown a little later, this relation holds closely for galvanometers from which the air is exhausted to a pressure of 0.2 mm of mercury.

plates of steel. I have never softened the wire at all, although I have heard that there is a slight advantage in tempering it. The needles are then cut off from the glass-hard wire by means of a simple shears, illustrated in the diagram (Fig. 2), which cuts all of nearly equal length. These short pieces, more in number than are actually required, are next measured under the microscope. Each piece is then stuck like a pin in the end of a little square of cardboard, on which is marked its length;

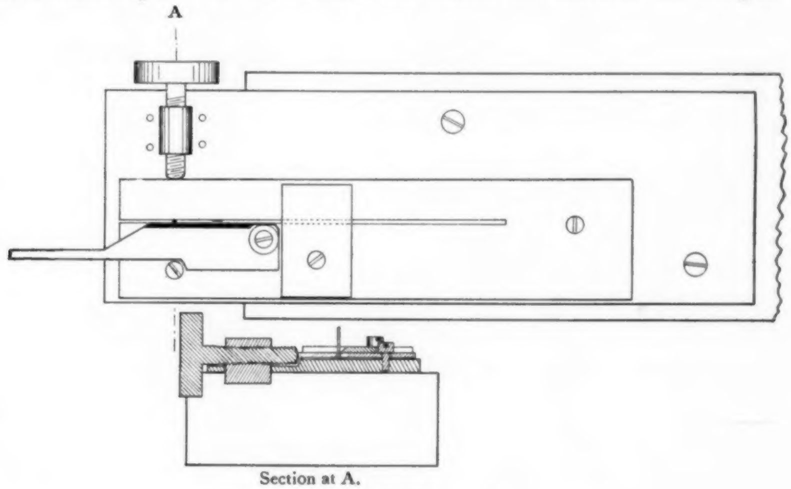


FIG. 2.

all the cardboards are put upon a rod and inserted together in a magnetizing helix, and the magnets are strongly magnetized to saturation. The magnetic moment of each is then determined and the weak ones are eliminated, and, from the remainder, the groups are selected so that total magnetic moment and total weight of all the groups shall be equal.

The groups are built up on a form shown in the accompanying diagram (Fig. 3), consisting of two straight-edged glass strips of equal thickness separated by a space of about 0.8 mm and stuck down to a piece of plate glass. In the crack between the straight edges runs a wire connected to binding posts, one of which is movable and connected to a spring so as to stretch the wire. The little magnets are laid in sugar syrup across the crack from edge to edge, and their distance apart is carefully

adjusted while the sugar is kept moist by breathing upon it; after which the syrup is allowed to harden. A straight glass stem is now drawn of the desired weight, and upon this at intervals corresponding with the distance between the groups of needles are placed little portions of shellac. The stem is now laid upon the needles and a proper current of electricity is passed through the wire underneath till it is seen with a hand glass that the shellac has run around every magnet, but without running out toward

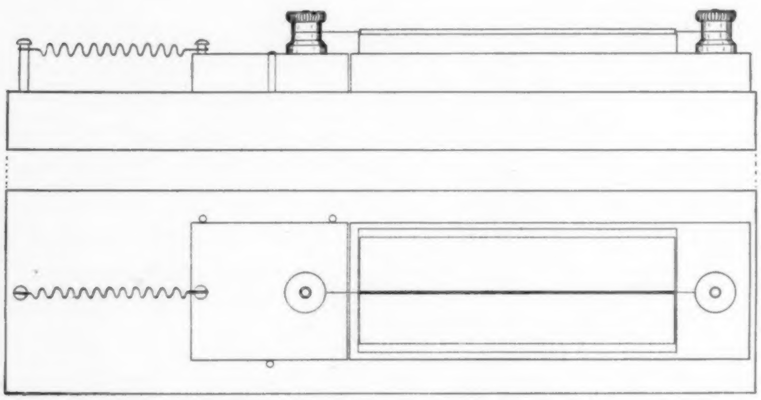


FIG. 3.

their ends. After the shellac cools, the needle system is dissolved off from the sugar bed with water, carefully washed, and is ready for the mirror. Such a needle system is almost mechanically perfect. I find no occasion with eight group needles to astaticize, but I do re-magnetize by the aid of an electro-magnet of sixteen poles.

The mirror is a fragment of microscope cover glass, selected from a large number of pieces of glass about 1 × 1.5 mm cut out with a dividing engine. These are all coated with metal on both sides, and their optical quality is tested by mounting each on a wad of cotton and reflecting with each mirror in succession the rays from a slit. Both surfaces are tested and each with both long sides and short sides vertical, so that the chance of obtaining a pretty good figure is quite considerable.

Galvanometer support and accessories.—In a city two well-known

kinds of disturbance are ever active, namely, mechanical jarring and electro-magnetic influences. The former of these may be communicated either by sound-waves in air or by direct ground vibrations. To remove the latter the Julius three-wire suspension is efficient, but may be combined advantageously with mercury flotation. Such a combination is employed for both the galvanometers at the Astrophysical Observatory, and its chief features will be found described in Vol. I of the Observatory *Annals*, and in the Appendix to the *Report of the Secretary of the Smithsonian Institution* for 1900 and 1901, pp. 100 and 122 respectively. I will merely add here that as a result of a series of experiments it was found best to use glycerine to damp the horizontal vibrations of the three-wire suspension, as this liquid appears to damp best in proportion to its transmission of ground tremors.

For the destruction of the disturbing vibrations of sound within the galvanometer case, and for a further purpose shortly to be spoken of, the most recent galvanometer case has been made air tight and is used at an air pressure of less than ½ mm of mercury.

Magnetic shielding.—Magnetic shielding is now employed for the most sensitive galvanometer. Three shields are made use of, whose dimensions are in general accord with the recommendations of Du Bois.[1] The outer one is a two-inch-thick cast-iron box open at top and bottom and about six feet high and three feet square. Within this hangs the heavy table upon which rests the galvanometer. Upon this table are two concentric soft cast-iron cylindrical shields. The outer is 30 cm high, 15 cm outside, and 13 cm inside diameter. The inner, symmetrically placed with regard to the outer and fastened to it, is 20 cm high, 8.7 cm in exterior, and 7.5 cm in inside diameter. Both shields are lifted up by means of a cord when one wishes to examine or adjust within the galvanometer. The latter is in the basement of the laboratory and the beam of light for reading it is brought down to the mirror from a slit 3 m above. A small side mirror within the galvanometer case at an angle of 45° to the vertical reflects

[1] *Wied. Ann.*, **65**, 8, 1898.

the beam upon the needle system, and returns it to the scale or photographic plate 4 m away in the room above.

A number of strong control magnets within the outer iron box outside the inner shield are used to adjust the time of swing and position of zero. As a constant temperature is essential to freedom from drift when the galvanometer is used at a high time of swing, automatic temperature regulation and other precautions are resorted to.

With this galvanometer with its relatively heavy needle system — protected so thoroughly from shaking by a mercury-floated Julius suspension system weighing nearly a ton, and from magnetic disturbances by three iron shields — great steadiness is of course obtained, and consequently it is practicable to use a high time of swing. But, as is well known, the damping of the air generally prevents the deflection of a modern galvanometer from increasing anything like the square of the time of swing, when the time of single swing is above 1 second, and the air has therefore been exhausted. It was found that little gain resulted till a pressure of less than 1 cm was reached, but at 0.8 mm mercury pressure the law of squares held to 2.5 seconds single swing, and at 0.2 mm the deflection remained proportional to the square of the time of swing up to a time of single swing of 5.5 seconds. The needle system was found to be almost surprisingly steady.

This brings the account down to September, 1902. Since then attention has been wholly diverted from the great galvanometer. It will illustrate the working sensitiveness of an instrument measuring currents of 1×10^{-12} ampères, when I say that the difference of radiating power between an observer's black coat distant 2 m from the balanced bolometer, and the walls of the room which his coat momentarily hid, threw the spot of light off the scale some 40 cm, while the observer's naked hand within a meter of the bolometer turned the galvanometer needle round and round.

In conclusion I wish to say that in the work above described I have had the co-operation of Mr. F. E. Fowle, Jr., of the Astrophysical Observatory.

WASHINGTON, D. C.,
May 1903.

ON FORMULÆ FOR SPECTRUM SERIES.

By A. FOWLER and H. SHAW.

IN spite of the admirable work on spectrum series which has been done by Rydberg and by Kayser and Runge, the formulæ which they have employed to express the relation between the lines which constitute a series can, in many cases, be regarded only as rough approximations, more particularly for series of the principal and second subordinate types. The present paper gives the results of an attempt to bring the results of calculation into better accordance with those of observation by employing modified formulæ.

After several trials, the most accurate representations of various series were obtained by further generalizing the modifications of Balmer's formula which have already been suggested or employed by others. Two of these formulæ seem to merit a somewhat detailed discussion.

Employing Rydberg's convenient notation so far as possible, the first may be written:

$$n = n_\infty - \frac{C}{m^p - m_0}, \tag{I}$$

where m takes successively the values 2, 3, 4, $\cdots$, n is the oscillation frequency corresponding to the different values of m, and n_∞, C, m_0, and p are four constants to be determined from four lines for which n and m are known; n_∞ is the "convergence frequency" of the series, or the value which n assumes when m is infinite. This formula may be considered as a variation of the generalization of Balmer's formula suggested by Ames,[1] which may be written

$$n = n_\infty - \frac{C}{m^2 - m_0},$$

in which m is taken to be a whole number.

The second equation investigated is

$$n = n_\infty - \frac{C}{(m+\mu)^2 - m_0}, \tag{II}$$

[1] *Phil. Mag.*, 30, 47, 1890.

where n_∞, C, μ, and m_0 are the four constants to be calculated for each series. This may be regarded as a combination of the formula of Ames with that of Rydberg,[1] the latter being

$$n = n_\infty - \frac{C}{(m+\mu)^2},$$

in which m is a whole number, but μ is usually a fraction, and C has the value 109675 for all series.

Formula II may be regarded geometrically as the equation to a hyperbola of the second degree with respect to axes parallel to the asymptotes, the co-ordinates being represented by the values of n and $(m+\mu)^2$, while n_∞ and m_0 are the co-ordinates of the point of intersection of the asymptotes. Similarly, Formula I may be looked upon as the equation to a hyperbola of the pth degree.

In terms of wave-lengths, the two formulæ may be written:

$$\lambda = \lambda_0 + \frac{C'}{m^p - m'}. \tag{Ia}$$

$$\lambda = \lambda_0 + \frac{C''}{(m+\mu)^2 - m''}. \tag{IIa}$$ [2]

DETERMINATION OF THE CONSTANTS.

The determination of the constants of the two formulæ is less tedious than might at first appear. Designating the four given lines by n_1, n_2, n_3, n_4, and the corresponding values of m by m_1, m_2, m_3, m_4, the index p in Formula I may be derived by a few trials from the relation

$$\frac{(n_4 - n_1)\ (n_3 - n_2)}{(n_2 - n_1)\ (n_4 - n_3)} = \frac{(m_4^p - m_1^p)\ (m_3^p - m_2^p)}{(m_2^p - m_1^p)\ (m_4^p - m_3^p)}.$$

If many series are to be investigated, it is convenient to construct curves showing the values of the expression on the right

[1] "Recherches sur la constitution des spectres d'émission des éléments chimiques," *Kgl. Svenska Vet.-Akad. Handl.*, **23**, 11, 1890.

[2] Since completing the present paper we have found that this formula has been previously employed by Rummel (*Proc. Roy. Soc. Victoria*, **10**, 75, 1897; **12**, 15, 1899), who, however, does not appear to have noticed that the equation is of the same form whether expressed in wave-lengths or frequencies.

The work of Rummel appears to have received far less attention than it deserves. This is perhaps partly due to the fact that in an earlier paper (*Proc. Roy. Soc. Vict.*, **9**, 260, 1897) the formula which he employed was simply that of Ames transposed to wave-lengths (which applies with reasonable accuracy to only a few series), and to his continued use of the symbol n (corresponding with m in the above formulæ) in his later papers for a number which may have a fractional value.

for various values of p and for different groups of values of m. From such curves the value of p may usually be read with a sufficient degree of accuracy, and when p has been determined the other constants are easily derived.

The value of $(m_1 + \mu)$ in Formula II is determined directly from the relation

$$(m_1 + \mu) = \sqrt{\frac{a}{a-3}} - 1.5 \ ,$$

where

$$a = \frac{(n_4 - n_1)\ (n_3 - n_2)}{(n_4 - n_3)\ (n_2 - n_1)} \ .$$

and n_1, n_2, n_3, n_4 refer to four consecutive lines. If four lines which are not consecutive are employed in the calculation of the constants, the above equation will not hold, but another of similar character may easily be obtained. When μ has been ascertained, the other constants readily follow; or, if desired, n_∞ may be determined independently.

In view of the possible physical significance of the term μ, it may eventually be desirable to adjust its value by the addition of an integer, when necessary, so that $m = 1$ always gives the first positive value of n, or, as in the case of hydrogen, makes $n = 0$. For the present this has not been considered necessary.

EXAMPLES.

As already remarked, it is chiefly in the case of series of the principal and second subordinate types that the older formulæ are most defective, and it may now be added that this is especially marked when it is attempted to include the least refrangible line of a series. The following examples illustrating the two formulæ under investigation have accordingly been chiefly selected from series of these types. In the first instance, the equations given are those calculated from the first four lines of each series.

Unless otherwise stated, the frequencies given in the tables have been derived from Kayser and Runge's wave-lengths (as given in Watts's *Index of Spectra*), corrected to vacuum by the revised table drawn up by the same observers.[1]

[1] *Abh. Königl. Preuss. Akad. Wissensch.*, Berlin, 1893; WATTS's *Index*, App. E, p. 52.

For the helium series, the frequencies already corrected to vacuum, are those given by Runge and Paschen.[1]

In the tables the numbers in brackets refer to the values of m; $O-C$ indicates observed minus computed values.

The differences $O-C$ in the case of Kayser and Runge's calculations, unless otherwise stated, are quoted from Professor Kayser's recent book (*Handbuch der Spectroscopie*, Vol. II), where the formulæ and tables are conveniently brought together.[2] Those resulting from the use of Rydberg's formula are taken directly from Rydberg's general work ("Recherches," etc.), but it should be pointed out that these would probably be slightly modified if recalculated with the later value of the constant C, namely 109675 in place of 109721.6.

THE PRINCIPAL SERIES OF SODIUM.

For the less refrangible components of the double lines which constitute the principal series of sodium lines, the equations derived from the first four lines are:

Formula I: $$n = 41510.37 - \frac{92513.4}{m^{1.917} - 0.02613}\ .$$

Formula II: $$n = 41449.92 - \frac{110875.4}{(m + 0.1935)^2 - 0.284956}\ .$$

OSCILLATION FREQUENCY IN VACUO	$O-C$ FORMULA I		$O-C$ FORMULA II		$O-C$ K. AND R.'s FORMULA (In Tenth-Meters)	$O-C$[4] K. AND R. (In Tenth-Meters)	$O-C$ RYDBERG (In Tenth-Meters)
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters			
16955.15	(2) +0.14	−0.049	(2) +0.03	−0.010	+77.46	+1.0	+3.0
30265.61	(3) −0.09	+0.010	(3) −0.01	+0.001	0.00	−1.6	−7.8
35041.12	(4) −0.03	+0.002	(4) 0.00	0.000	0.00	+1.0	+0.7
37295.34	(5) −0.02	+0.001	(5) 0.00	0.000	0.00	+0.7	+1.2
38539.66[3]	(6) −0.04	+0.002	(6) +1.81	−0.121	+ 0.06	+0.1	+2.9
39299.06[3]	(7) −1.15	+0.074	(7) +3.67	−0.237	+ 0.17	−0.4	.....
39793.70[3]	(8) −5.90	+0.370	(8) +2.39	−0.160	+ 0.65	−0.4	.....

[1] *Sitz. d. K. Akad. der Wiss.*, Berlin, July, 1895, pp. 639, 759; ASTROPHYSICAL JOURNAL, January 1896.

[2] The equation generally employed by Kayser and Runge is $N = A\ B\frac{\beta}{m^2} - \frac{C}{m^4}$, where A, B, C, are three constants to be separately determined for each series, and m is a whole number.

[3] Observed as single lines.

[4] The errors in this column are those resulting from a formula in which all the lines are used in the calculation of constants (KAYSER, *Handbuch der Spect.*, Vol. II, p.521).

PRINCIPAL SERIES OF POTASSIUM.

For the less refrangible components of the principal series of double lines of potassium the formulæ and results are as follows:

$$\text{Formula I: } n = 34927.31 - \frac{195630.4}{m^{2.2182} - 2.5224} \; .$$

$$\text{Formula II: } n = 35047.96 - \frac{115321.9}{(m + 0.459)^2 - 0.81980} \; .$$

Oscillation Frequency in Vacuo	*O–C* Formula I In Frequency	*O–C* Formula I In Tenth-Meters	*O–C* Formula II In Frequency	*O–C* Formula II In Tenth-Meters	*O–C* K. & R.'s Formula In Tenth-Meters	*O–C* Rydberg In Tenth-Meters
12984.69	(3) 0.00	0.000	(2) −0.03	+0.020	+161.30	−17.0
24700.46	(4) −0.01	+0.002	(3) +0.02	−0.003	0.00	−22.7
28998.41	(5) 0.00	0.000	(4) 0.00	0.000	0.00	−6.6
31068.72	(6) 0.00	0.000	(5) 0.00	0.000	0.00	+1.6
32224.22	(7) −0.91	+0.087	(6) −4.06	+0.390	+0.27	+4.0
32940.18	(8) +4.53	−0.417	(7) −4.01	+0.369	+0.23	+4.3
33409.17	(9) +6.58	−0.588	(8) −8.45	+0.755	+0.68	+6.6
33736.46	(10)+11.18	−0.978	(9)−10.67	+0.934	+1.05	+7.3
33971.45	(11)+14.24	−1.234	(10)−14.33	+1.242	+1.45	+6.7

THE PRINCIPAL SERIES OF LITHIUM.

For this series the formulæ and results are as follows:

$$\text{Formula I: } n = 43446.07 - \frac{116717.5}{m^{2.032} - 0.00056} \; .$$

$$\text{Formula II: } n = 43470.35 - \frac{108764.1}{(m + 0.931)^2 + 0.07855} \; .$$

Oscillation Frequency in Vacuo	*O–C* Formula I In Frequency	*O–C* Formula I In Tenth-Meters	*O–C* Formula II In Frequency	*O–C* Formula II In Tenth-Meters	*O–C* K. & R. In Tenth-Meters	*O–C* K. & R.[2] In Tenth-Meters	*O–C* Rydberg In Tenth-Meters
14903.08	(2)+0.25	−0.111	(1)−0.09	+0.040	+108.12	0.00	+0.4
30924.54	(3)−0.16	+0.017	(2)+0.06	−0.006	0.00	0.00	+2.2
36467.48	(4)−0.05	+0.004	(3)+0.02	−0.001	0.00	0.00	+0.5
39011.60	(5)−0.03	+0.002	(4)+0.02	−0.001	0.00	+0.66	−0.8
40389.99	(6)+5.45	−0.350	(5)+4.68	−0.287	−0.20	+0.95	−0.3
41215.53	(7)+7.77	−0.457	(6)+5.57	−0.327	−0.01	+1.44	−0.4
41749.29	(8)+9.54	−0.550	(7)+5.92	−0.338	+0.29	+1.80	−0.4
42112.35[1]	(9)+9.41	−0.530	(8)+4.25	−0.238	+0.75	+2.18	−0.6

[1] Based on observations by Messrs. Liveing and Dewar.

[2] The errors in this column are those which result when the constants are calculated from the first three lines.

THE SERIES OF MAGNESIUM TRIPLETS.

For the middle components of the triplets which constitute the first subordinate series of magnesium lines the following formulæ have been derived from the first four lines of the series:

$$\text{Formula I: } n = 39762.68 - \frac{126484.3}{m^{2.058} - 0.34427} \,.$$

$$\text{Formula II: } n = 39790.41 - \frac{109684.5}{(m + 0.840)^2 - 0.06227} \,.$$

Oscillation Frequency in Vacuo	O–C Formula I		O–C Formula II		O–C K. & R.'s Formula In Tenth-Meters	O–C Rydberg's Formula In Tenth-Meters
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters		
26085.57	(3) 0.00	0.000	(2)+0.02	−0.003	−0.01	+0.5
32320.40	(4)+0.01	−0.001	(3)−0.01	+0.001	−0.01	−1.1
35095.68	(5) 0.00	0.000	(4) 0.00	0.000	+0.31	−0.1
36568.50	(6)+0.01	−0.001	(5) 0.00	0.000	0.00	−0.6
37444.55	(7)+2.29	−0.162	(6)+1.67	−0.118	−0.41	+0.9
38004.23	(8)+1.71	−0.118	(7)+0.12	−0.008	−0.46	+0.4

For the least refrangible components of the second subordinate series of triplets the formulæ are:

$$\text{Formula I: } n = 39699.68 - \frac{176665.9}{m^{2.175} - 2.25379} \,.$$

$$\text{Formula II: } n = 39794.75 - \frac{115447.6}{(m + 0.555)^2 - 0.89949} \,.$$

Oscillation Frequency	O – C Formula I		O – C Formula II		O – C K. & R.'s Formula (In Tenth-Meters)	O – C Rydberg's Formula (In Tenth-Meters)
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters		
($b1$) 19285.44	(3) 0.00	0.000	(2)+0.03	−0.008	+53.22	+3.9
29960.21	(4)−0.01	+0.001	(3)−0.02	+0.002	−0.03	−13.0
33978.46	(5)−0.01	+0.001	(4)+0.01	+0.001	−0.05	−2.2
35941.23	(6)+0.04	−0.003	(5) 0.00	0.000	−0.02	+2.5
37047.87	(7)−0.18	+0.013	(6)−2.64	+0.198	+0.11	+3.8
37734.99	(8)+1.80	−0.127	(7)−4.76	+0.336	+0.27	+4.8

RYDBERG'S SERIES OF MAGNESIUM.

There is another series of magnesium lines to which attention was first drawn by Rydberg[1] for which the formulæ of Rydberg

[1] *Öfversigt af Kongl. Vet.-Akad. Forhandl.* Stockholm, 1893.

and of Kayser and Runge give very unsatisfactory results. For this series Rydberg has employed the modified formula:

$$n = n_{\infty} - \frac{B}{(m+\mu)^2} - \frac{C}{(m+\mu)^4} .$$

It is interesting to find that the new formulæ may be applied to the series without change. As the lines are somewhat nebulous, it is possible that the wave-lengths of the more refrangible members of the series, which are also feeble lines, are not very precise, and in fact for the last two lines the limit of error stated by Kayser and Runge is 1.0 tenth-meter.

Formula I: $n = 26633.03 - \frac{88847.1}{m^{1.925} + 2.10247}$.

Formula II: $n = 26601.49 - \frac{107071.4}{(m + 0.2304)^2 + 2.13282}$.

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C$ Rydberg's Modified Formula
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	
18082.33	(3) −0.01	+0.003	(3) 0.00	0.00	+0.019
21255.70	(4) 0.00	0.000	(4) 0.00	0.00	−0.009
22970.71	(5) 0.00	0.000	(5) 0.00	0.00	−0.004
23986.85	(6) −0.01	+0.002	(6) 0.00	0.00	0.000
24633.15	(7) −1.01	+0.170	(7) −0.53	+0.09	+0.200
25074.10	(8) +3.61	−0.570	(8) +5.00	−0.79	−0.480

THE TWO SUBORDINATE SERIES OF HELIUM DOUBLES.

For the less refrangible components of the first subordinate series, the formulæ and results are as follows:

Formula I: $n = 29222.11 - \frac{110093.6}{m^{2.0015} + 0.00332}$.

Formula II: $n = 29222.89 - \frac{109697.9}{(m + 0.996)^2 + 0.009151}$.

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C$[1] K. & R.'s Formula		$O-C$[2] Rydberg's Formula	
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters
D_3 17014.13	(3) 0.00	0.000	(2)+0.02	−0.007	0.00	0.00	−9.43	+3.263
22356.97	(4) 0.00	0.000	(3)−0.01	+0.002	0.00	0.00	+2.40	−0.480
24829.56	(5) 0.00	0.000	(4) 0.00	0.000	0.00	0.00	+0.74	−0.119
26172.44	(6) 0.00	0.000	(5) 0.00	0.000	−0.13	+0.02	+0.25	−0.036
26981.94	(7)−0.08	+0.011	(6)−0.10	+0.014	−0.39	+0.05	−0.01	+0.001
27507.23	(8)−0.11	+0.013	(7)−0.16	+0.021	−0.60	+0.08	−0.12	+0.014
27867.34	(9)−0.12	+0.015	(8)−0.20	+0.026	−0.78	+0.10	−0.17	+0.022
28124.73	(10)−0.02	+0.003	(9)−0.43	+0.054	−1.09	+0.14	−0.36	+0.046
28315.49	(11)−0.05	+0.006	(10)−0.21	+0.026	−1.00	+0.13	−0.16	+0.020
28460.56	(12)+0.29	−0.045	(11)−0.08	+0.010	−0.94	+0.12	−0.01	+0.001
28573.37	(13)+0.19	−0.023	(12)−0.06	+0.007	−0.99	+0.12	+0.03	−0.004
28662.74	(14)+0.10	−0.012	(13)−0.17	+0.021	−1.16	+0.14	−0.08	+0.010
28735.00	(15)+0.20	−0.024	(14)−0.10	+0.012	−1.14	+0.14	0.00	0.000
28794.33	(16)+0.48	−0.057	(15)+0.15	−0.018	−0.93	+0.11	+0.26	−0.031
28843.27	(17)+0.50	−0.060	(16)+0.12	−0.014	−0.99	+0.12	+0.24	−0.029
28881.93	(18)−1.86	+0.222	(17)−2.25	+0.268	−3.39	+0.41	−2.12	+0.253
28919.43	(19)+0.94	−0.112	(18)+0.53	−0.063	−0.64	+0.08	+0.66	−0.079

For the second subordinate series the formulæ for the less refrangible components, as calculated from the first four lines, are as follows:

$$\text{Formula I:}\quad n = 29176.95 - \frac{142019.0}{m^{2.103} - 0.62765}\,.$$

$$\text{Formula II:}\quad n = 29227.70 - \frac{110377.6}{(m + 0.724)^2 - 0.099836}\,.$$

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C$ K. & R.'s Formula		$O-C$[4] Rydberg's Formula	
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters
14149.48	(3)+ 0.01	−0.005	(2)−0.02	+0.011	−47.85	+24.00	−44.86	+22.660
21210.94	(4) 0.00	0.000	(3)+0.01	−0.002	0.00	0.00	− 6.70	+ 1.500
24259.39	(5) 0.00	0.000	(4)−0.01	+0.002	0.00	0.00	− 1.41	+ 0.241
25848.55	(6) 0.00	0.000	(5) 0.00	0.000	0.00	0.00	− 0.16	+ 0.024
26780.61	(7)+ 0.84	−0.117	(6)−0.36	+0.050	+ 1.38	− 0.19	+ 0.11	− 0.015
27373.71	(8)+ 1.27	−0.168	(7)−0.78	+0.103	+ 3.44	− 0.46	+ 0.18	− 0.024
27774.08	(9)+ 4.04	−0.524	(8)−1.44	+0.187	+ 5.37	− 0.70	− 0.03	+ 0.004
28057.39	(10)+ 6.35	−0.807	(9)−1.75	+0.222	+ 7.51	− 0.95	+ 0.05	− 0.006
28264.92	(11)+ 8.55	−1.064	(10)−2.40	+0.299	+ 9.28	− 1.16	− 0.03	+ 0.004
28421.47	(12)+10.64	−1.324	(11)−2.62	+0.326	+10.76	− 1.33	− 0.17	+ 0.024
28543.27	(13)+13.41	−1.634	(12)−2.25	+0.274	+12.82	− 1.57	+ 0.47	− 0.057
28638.93	(14)+15.31	−1.865	(13)−2.43	+0.296	+14.10	− 1.72	+ 0.52	− 0.063
28714.37[3]	(15)+15.99	−1.948	(14)−3.97	+0.484	+13.85	− 1.69	− 0.82	+ 0.100

[1] For all the helium series Runge and Paschen use the formula:

$$n = A - \frac{B}{m^2} - \frac{C}{m^3}\,.$$

[2] Rydberg does not appear to have published calculations for the helium lines, but he has published statements of the convergence frequencies of the series (ASTROPHYSICAL JOURNAL, **4**, 94, 1896) and of the values of the term μ in his equation (*Ibid.*, **6**, 234, 1897). By making use of these we get for this series the formula

$$n = 29222.70 - \frac{109675}{(m + 0.996084)^2}\,,$$

and it is from this that the tabulated errors have been derived.

[3] Marked "doubtful" in Runge and Paschen's table.

[4] See note to previous table; the errors stated have been calculated from the equation

$$n = 29222.63 - \frac{109675}{(m + 0.701464)^2}\,.$$

GENERAL DISCUSSION.

The foregoing examples suffice to demonstrate that series which present great departures from calculation when the older formulæ are employed can be better represented by either of the two formulæ investigated, but few of them permit a final comparison of their relative merits or of the ultimate accuracy of which they are capable. In some of the series there are probably small errors of observation which account in part for differences between the observed and calculated frequencies, and in most of them comparatively few lines have been traced. For helium, however, the case is different; many lines have been observed, and as they are also well defined, the positions determined by Runge and Paschen are probably accurate to a high degree.

All the helium series are not, however, well adapted to test the different formulæ. The first subordinate series of doubles, for example, is very closely represented even by the simplest generalization of Balmer's formula for hydrogen, the equation calculated from lines for which m has the values 3, 4, 5, being

$$n = 29225.58 - \frac{109901.37}{m^2} .$$

With this formula the differences between the observed and calculated frequencies for values of m as far as 17 do not differ by more than 2.13. As all the formulæ under investigation are attempted generalizations of Balmer's law, it is clear that such a series as this cannot be effectively employed for their comparison, and it will be seen by reference to the table that the differences between the observed and calculated frequencies are very small throughout. It is the second subordinate series of double lines of helium which at present permits the best comparison of the formulæ.

From the table already given for this series it appears that Formula I, when calculated for the first four lines, gives errors comparable with those of Kayser and Runge's formula for lines corresponding to the greater values of m, but has the advantage of correcting the large error in the case of the first line. Still, this cannot be considered an accurate formula for the series,

especially as the convergence frequency is notably smaller than that derived from the first subordinate series. Nevertheless it is instructive to observe the effect of calculating the constants from lines more widely separated than the first four; taking lines for which $m = 3, 4, 6$, and 8, we get equation I*b* stated at the head of the following table. Again, reducing p to 2.09 and calculating the other constants from the first three lines, we get equation I*c*.

The results are as follows:

Formula I*a*: $$n = 29176.95 - \frac{142019.0}{m^{2.103} - 0.62765}.$$

Formula I*b*: $$n = 29189.83 - \frac{140337.9}{m^{2.0947} - 0.65602}.$$

Formula I*c*: $$n = 29200.77 - \frac{139489.5}{m^{2.09} - 0.66775}.$$

Oscillation Frequency in Vacuo	$O-C$ Formula I*a* In Frequency	$O-C$ Formula I*b* In Frequency	$O-C$ Formula I*c* In Frequency
14149.48	+0.01[1]	−0.02[1]	0.00[1]
21210.94	0.00[1]	+0.02[1]	0.00[1]
24259.39	0.00[1]	+0.61	0.00[1]
25848.55	0.00[1]	−0.01[1]	−1.66
26780.61	+0.84	−0.36	−3.13
27373.71	+1.27	0.00[1]	−3.76
27774.08	+4.04	−1.62	−3.96
28057.39	+6.35	+1.98	−3.38
28264.92	+8.55	+3.31	−2.67
28421.47	+10.64	+4.64	−1.86
28543.27	+13.41	+6.67	−0.20
28638.93	+15.31	+8.23	+0.89
28714.37	+15.99	+8.26	+0.59

The errors may in this way be generally reduced and differently distributed, but it is clear that the series cannot be represented by Formula I with an accuracy equal to the probable accuracy of the observations.

For the same series, Formula II, as derived from the first four lines, is decidedly more accurate than Formula I derived from the same four lines, and the convergence frequency is also very nearly equal to that of the first subordinate series. The regular increase in the differences $O - C$ suggests that the formula might

[1] Used in calculation of constants.

be improved by taking a value of μ slightly different from that given by the first four lines. Thus, by calculating the constants from lines for which $m = 2, 3, 5$, and 7, the value of μ becomes 0.716, and the equation derived is

$$n = 29224.24 - \frac{110040.6}{(m + 0.716)^2 - 0.07766} .$$

Here the convergence frequency differs very slightly from that of the first subordinate series, and the differences $O - C$ are greatly reduced, as shown in the following table:

Observed Frequency in Vacuo	$O - C$ Formula II Above	
	In Frequency	In Tenth-Meters
14149.48	−0.01	+0.005
21210.94	0.00	0.000
24259.39	−0.24	0.041
25848.55	0.00	0.000
26780.61	+0.03	−0.004
27373.71	0.00	0.000
27774.08	−0.28	+0.036
28057.39	−0.32	+0.041
28264.92	−0.49	+0.061
28421.47	−0.72	+0.091
28543.27	−0.17	+0.021
28638.93	−0.20	+0.024
28714.37	−1.61	+0.196

The differences between calculation and observation thus become almost insignificant, never amounting to more than 0.1 tenth-meter, except in the case of the last line, which is marked "doubtful" by Kayser and Runge.

Of the other examples given, the principal series of potassium is about the only one which is sufficiently extended to be used in a minute comparison of the formulæ, but in this case the wave-length determinations are probably much less precise than those of helium. From the table already given it will be seen that when the constants are determined from the first four lines, the two formulæ investigated in this paper represent the series with almost equal accuracy, and much more closely than the older formulæ. As in the case of the second subordinate series of helium, however, it is found that changes in the constants of Formula I do not result in quite so accurate a representation of

the series as can be obtained by a change of constants in Formula II. Thus by calculating the constants of the latter from the first, second, fourth, and six lines, we get the equation:

$$n = 35031.60 - \frac{113919.3}{(m + 0.430)^2 - 0.7380} ;$$

and the differences $O - C$ compared with the limits of error of observation stated by Kayser and Runge are as follows:

Observed Frequency in Vacuo	$O-C$ Formula Above		Limit of Error of Observation in Tenth-Meters
	In Frequency	In Tenth-Meters	
12984.69	+0.99	−0.58	5.00
24700.46	−0.06	+0.01	0.03
28998.41	−1.59	+0.19	0.03
31068.72	−0.04	+0.004	0.03
32224.22	−1.96	+0.19	0.10
32940.18	+0.11	−0.01	0.10
33409.17	−2.58	+0.23	0.15
33736.46	−3.35	+0.29	0.20
33971.45	−5.80	+0.50	1.00

When the errors are distributed in this way, it will be seen that they are on the whole scarcely too great to be considered as arising from errors of observation.

The general result of the comparison, therefore, is that, while both the formulæ investigated represent the observed series very closely, the most accurate general formula appears to be:

$$n = n_\infty - \frac{C}{(m + \mu)^2 - m_0} .$$

With this formula, if the constants are calculated so as to distribute the errors, the computed differ from the observed positions by amounts which perhaps do not exceed the probable errors of the observations at present available.

Royal College of Science, London,
March 30, 1903.

THE VARIABLE STAR 6871 *V LYRAE*.

By J. A. Parkhurst.

The variability of this star was announced by Anderson in 1895[1], this being one of many variables which are very faint for the greater part of their period, and yet have been detected by this careful observer with telescopes of two and three inches aperture. In the place just cited, magnitudes are given from the Harvard photographs from July 24, 1892, to October 6, 1894. Visual observations have been published by Yendell,[2] H. M. Parkhurst,[3] and the writer,[4] covering the maxima of each year from 1893 to 1900, inclusive. Approximate data for the minima of 1899 and 1900 are given in the last reference. Photometric magnitudes of comparison stars brighter than the twelfth magnitude have also been published by H. M. Parkhurst,[5] with which those given in this paper are in fair agreement.

INSTRUMENTS.

These are the same as given in my paper on *X Cephei*[6]—a 157 mm reflector, a 305 mm, and the 101 cm refractor of the Yerkes Observatory; the equalizing wedge photometer being attached to each of these instruments. The photograph of the field reproduced in Plate I was taken November 30, 1902, with an exposure of thirty-six minutes, from $6^h\ 24^m$ to $7^h\ 0^m$ Central Standard Time. The faintest stars shown on the plate are about 16.5 magnitude, though none fainter than ϵ, 15.80 magnitude, have been measured in this field.

POSITION OF THE VARIABLE.

The variable was connected on two nights with the comparison stars *a*, *b*, and *c*, whose *Durchmusterung* numbers will be

[1] *A. N.*, **137**, 235.

[2] *Astronomical Journal*, **15**, 157, 1895; **17**, 27, 1896.

[3] *Ibid.*, **17**, 65, 1897; **18**, 100, 1897; **19**, 190, 1899.

[4] *Ibid.*, **17**, 102, 1897; **18**, 142, 1898; Astrophysical Journal, **14**, 174, 1901.

[5] *Ibid.*, **17**, 65, 1897.

[6] Astrophysical Journal, **17**, 48, 1903.

found in Table II. The places of these stars for 1875 were taken from the Cambridge, England, *Astronomische Gesellschaft* Catalogue. The results were as follows:

STAR	*a*			*b*		*c*	
R.A. 1875	19^h	3^m	$1^s.02$	4^m	$8^s.51$	4^m	$10^s.80$
ΔR.A.		+1	9.93	+0	3.27	+0	0.30
R.A. *V*	19	4	10.95	4	11.78	4	11.10
Mean *V* 1875	19	4	11.28				
Dec., 1875	+29°	36′	16″.2	42′	43″.9	17′	48″.7
Δ Dec.		−8	42.4	−15	10.5	+9	43.3
V	+29	27	33.8	27	33.4	27	32.0
Mean *V* 1875	+29	27	33.1				

For convenience the following are added:

POSITION OF 6871 *V LYRAE*

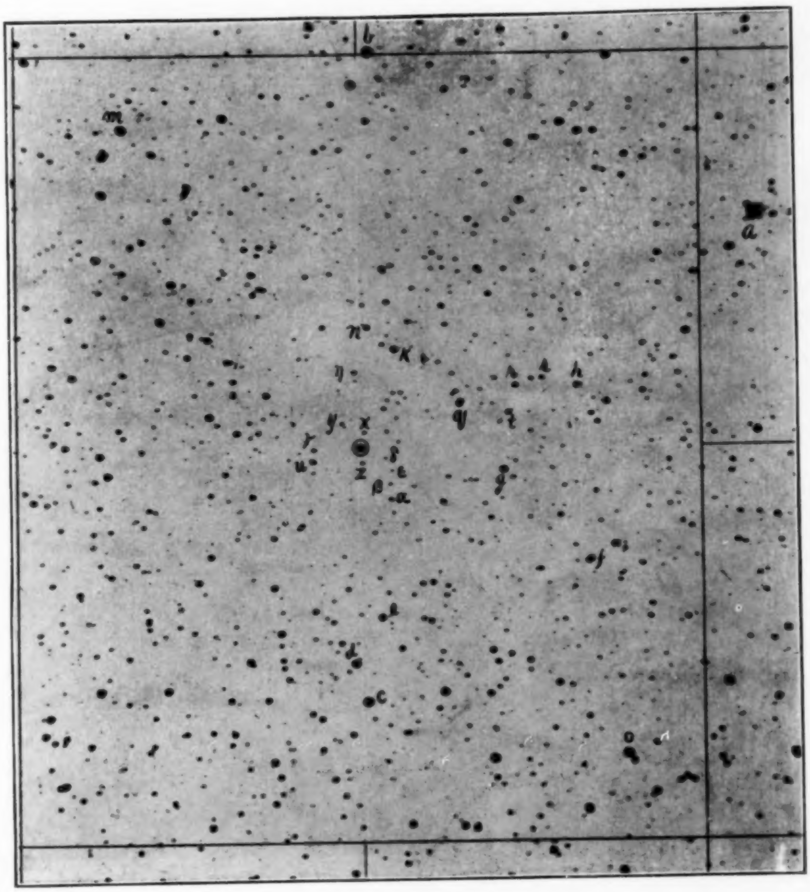
R.A. 19^h 3^m $24^s.5$, Dec. +29° 25′ 42″ (1855).
5 9.9, 29 52 (1900).

COMPARISON STARS.

The numerical data for the comparison stars are given in Tables I and II, which require little explanation. The positions of all the stars are shown on the chart. The co-ordinates from the variable for the stars brighter than twelfth magnitude were measured with the filar micrometer on the 6-inch reflector; the faint stars near the variable were measured with the 40-inch refractor, and all positions were checked from the photograph. The light scale was formed in the usual manner, from comparisons of the variable by Argelander's method; the results are given in the fourth column. The zero point of this scale, the star ϵ, is not the limit of vision of the 40-inch refractor, but simply the faintest star needed in the visual comparisons. The fifth column shows the results of the photometer measures which are given in detail in Table III.

The photometric magnitudes are based on the following standard stars:

PLATE I.



6871 *V LYRAE*

($19^h\ 5^m\ 9^s.9$; $+29°\ 29'\ 52''$)

Star	B.D.		1855		Magnitudes		
	No.	Mag.	R.A.	Dec.	*P.DM.*	Harvard 24	Harvard 45
A	+29° 3472	6.5	$19^h\ 0^m\ 7^s.6$	+29° 41′.9	6.45	6.46	6.77
B	+30 3425	7.8	1 54.4	+30 1.0	....	8.06	....
C	+30 3438	7.0	3 14.7	+30 4.4	6.94	6.86	6.90

The magnitudes in the last three columns are, respectively, from the *Potsdam Photometric Durchmusterung*, *Harvard College Observatory*, Vol. 24 (*Meridian Photometry*), and Vol. 45 (*Photometric Durchmusterung*). The magnitudes used were those of Harvard 24, as it contained all three of the stars and differed but little from the *P.DM*, the difference in the systems being only 0.035 mag. In the sixth column of Table I are given the magnitudes of such of the stars as were not measured with the photometer. These were read off from the "Magnitude Curve," Fig. 1, in a manner to be described later.

TABLE I.

Comparison Stars for *V Lyrae*. In order of Right Ascension.

Star	Co-ordinates from *V*: R. A.	Co-ordinates from *V*: Dec.	Light Scale	Magnitude: Measured	Magnitude: From Curve	Star	Co-ordinates from *V*: R. A.	Co-ordinates from *V*: Dec.	Light Scale	Magnitude: Measured	Magnitude: From Curve
	s	′ ″	steps				s	′ ″	steps		
a	−70	+ 8 43	....	8.29	.....	δ	− 4	− 0 25	1.5	15.60	.....
o	−45	−11 54	37.6	9.35	.....	*e*	− 3	− 6 26	29.2	.....	11.27
f	−44	− 3 49	28.1	11.18	.....	*b*	− 2	+15 11	44.3	8.54	.....
h	−38	+ 2 24	28.4	.....	11.40	*n*	− 2	+ 4 36	26.0	12.20	.....
s	−31	+ 2 40	....	.....	.....	*x*	− 1	+ 0 35	....	.....	14.8
r	−27	+ 2 25	19.0	.....	12.88	*c*	− 1	− 9 43	35.9	10.05	.....
t	−26	+ 1 18	....	.....	.....	*z*	− 1	− 0 34	8.0	14.76	.....
g	−25	− 0 52	25.4	11.22	.....	η	+ 1	+ 2 54	....	13.35	.....
q	−17	+ 1 42	23.0	.....	12.30	*d*	+ 1	− 8 8	31.8	.....	10.80
l	− 7	+17 48	...	.....	.....	*y*	+ 3	+ 0 59	....	.....	.....
k	− 5	+ 3 49	27.0	11.91	.....	γ	+ 8	− 0 3	7.7	.....	14.70
ε	− 6	− 0 50	0.0	.....	15.80	*u*	+ 8	− 0 29	16.4	13.08	.....
α	− 5	− 2 00	15.3	.....	13.52	*m*	+40	+12 00	37.0	9.81	.....
β	− 5	− 1 21	7.1	.....	14.78						

Table II gives the *Durchmusterung* numbers of part of the stars, also a comparison between the *Durchmusterung* magnitudes and the photometric measures of H. M. Parkhurst and the writer. There is a systematic difference of about 0.15 mag.

between the two latter, which can be explained by the fact that they are based on a different set of Meridian Photometer stars. Besides this, there is a difference of 0.60 mag. in the star *b*, which has a range of over a magnitude from the *Durchmusterung* value. The photographic magnitude of *b* is about 9.2, showing a considerable color and probable fluctuations or variation.

TABLE II.

Comparison with the *Durchmusterung* and Measures by H. M. Parkhurst.

Star	"Durchmusterung"		H. M. Parkhurst		J.A. Parkhurst
	Number	Mag.	Letter	Mag.	
a	+29° 3483	8.0			8.29
o	3486	9.2			9.35
l	3490	8.4			....
b	3492	8.1	*J*	9.14	8.54
c	3493	9.3	*IX*	9.91	10.05
d	3494	9.5	*Z*	10.70	10.80
m	+29 3498	9.2	*W*	9.67	9.81
f	....	...	*b*	10.97	11.18
h	....	...	*c*	11.02	11.40
g	....	...	*a*	10.95	11.22
e	....	...	*e*	11.28	11.27

The photometric measures of the comparison stars are given in Table III. The arrangement of this table and the methods of reduction are similar to those in my previous paper on *X Cephei*. For convenience I will repeat here that the stars used as standards are given first in each part of the table, with their magnitudes in bold-faced type. The mean of the scale readings and the mean of the magnitudes of these standard stars are taken as the zero points. For the stars whose magnitudes are sought, the difference in scale readings is converted into magnitudes by the use of the wedge constant given at the foot of each set, and added to the zero magnitude, giving the quantities in the last column. The magnitudes of the comparison stars brighter than 11.5 were measured with the 6- and 12-inch telescopes, and the mean of the results given after the set for October 5, 1902. From these as standards, the stars *n* and *k* were measured with the 12-inch, and finally the four fainter stars were measured with

the 40-inch. The set of measures made October 5, 1902, requires explanation, since a cell holding two absorption glasses was interposed in the cone of rays of the real star while measuring the standards *A*, *B*, and *C*. The absorption caused by these two glasses has been measured by standard stars and also by the "wheel photometer," the resulting value, 1.70 mag., being used in this work. The mean magnitude of the three standard stars, 7.13, becomes 8.83 when the absorption glasses are used. It will be seen that the results are in good accord with those taken without the absorption glasses.

TABLE III.

1900, August 4; 6-inch. Wedge II.

Star	Scale Readings								Mean Scale Reading	Magnitude
	First				Second					
A	11.8	10.5	10.5	8.8	12.2	9.9	10.9	11.0	10.60	**6.46**
B	20.0	20.9	21.2	20.1	22.2	22.0	20.9	21.5	21.12	**8.06**
C	15.0	14.1	14.8	14.4	14.9	13.1	14.2	13.1	14.20	**6.86**
a	25.2	23.6	24.2	23.9	23.6	22.8	23.8	22.0	23.64	8.21
b	25.0	25.0	25.3	25.2	24.2	24.6	24.7	25.9	24.99	8.39
m	37.2	35.1	36.1	35.7	34.1	33.9	34.6	33.6	35.04	9.70
c	34.4	36.3	37.0	35.9	37.2	38.9	37.7	37.9	37.04	9.96
o	32.9	29.2	30.9	32.6	32.4	31.9	31.9	31.9	31.72	9.26

Wedge constant, 0.130 mag.

1900, October 13; 6-inch. Wedge II; seeing fine.

C	12.2	10.0	9.5	9.8	8.8	8.2	9.75	**6.86**
B	17.8	18.8	19.5	17.1	17.2	17.3	17.95	**8.06**
a	20.1	21.1	20.9	21.0	21.2	21.9	21.04	8.39
b	23.9	23.5	24.2	22.5	24.3	22.1	23.42	8.70
m	33.5	32.6	33.0	32.3	32.2	32.6	32.70	9.91
c	35.0	34.8	34.7	33.0	34.0	35.0	34.42	10.13
o	29.1	28.3	28.8	28.0	29.9	30.1	29.03	9.43
g	42.9	45.1	43.5	....	....	....	43.83	11.36
f	42.0	43.2	43.3	....	....	....	42.50	11.18
k	48.5	50.7	50.2	....	....	....	49.80	12.13
v	36.7	39.3	37.5	....	....	....	37.83	10.38

Wedge constant, 0.130 mag.

TABLE III—*Continued.*

1902, October 5; 12-inch. Wedge V; seeing good.

C_a	12.0	13.1	11.8	12.3	13.0	12.7	12.49	**6.86**
B_a	17.8	18.5	18.2	21.0	20.5	19.9	19.32	**8.06**
A_a	9.2	9.1	9.1	10.0	9.1	8.8	9.22	**6.46**
g	35.0	34.4	35.7	33.0	33.0	33.3	34.07	11.07
q	41.2	42.8	42.4	43.3	42.0	41.8	42.25	11.97
k	40.7	40.0	40.9	38.5	38.3	38.9	39.55	11.68
n	43.1	44.9	44.0	41.3	40.5	40.3	42.35	11.98
b	10.6	10.3	11.0	11.2	11.8	11.2	11.02	8.54
a	8.8	9.8	8.3	8.8	8.7	7.8	8.70	8.28
v	49.9	50.2	49.8	49.1	48.7	48.0	49.29	12.75

Wedge constant, 0.110 mag.

RESULTING MAGNITUDES FROM MEASURES WITH 6- AND 12-INCH.

Star	1900 Aug. 4	1900 Oct. 13	1902 Oct. 5	Mean
a	8.21	8.39	8.28	8.29
b	8.39	8.70	8.54	8.54
o	9.26	9.43	.....	9.35
m	9.70	9.91	.....	9.81
c	9.96	10.13	.....	10.05
f	....	11.18	.....	11.18
g	....	11.36	11.07	11.22

1900, October 17; 12-inch. Wedge II; seeing fair.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
b	20.4	19.6	19.9	20.9	20.5	19.5	20.14	**8.54**
m	31.7	29.3	29.0	32.7	30.0	30.6	30.55	**9.81**
c	33.1	33.9	32.2	35.0	34.1	35.0	33.98	**10.05**
o	25.0	25.1	24.7	24.8	23.8	25.0	24.73	**9.35**
n	48.9	49.6	47.8	51.3	51.0	49.8	49.90	12.37
k	45.5	46.7	46.3	46.8	46.2	43.8	45.89	11.83
u	61.5	59.8	62.1	....	....	....	61.13	13.83
v	35.7	35.4	35.1	....	....	....	35.40	10.48

Wedge constant, 0.130 mag.

TABLE III—*Continued.*

1902, May 14; 12-inch. Wedge V; seeing good.

a	14.5	15.2	15.1	15.0	15.2	15.7	15.12	**8.29**
b	18.8	18.8	18.9	17.5	17.7	17.7	18.23	**8.54**
m	29.9	30.1	30.1	29.0	29.2	29.9	29.70	**9.81**
c	35.3	35.4	36.3	35.0	35.0	34.2	35.20	**10.05**
n	52.8	53.4	52.8	51.9	52.5	52.3	52.62	12.26
k	50.7	51.2	50.1	49.2	49.5	50.1	50.14	11.98

Wedge constant, 0.110 mag.

RESULTING MAGNITUDES FROM MEASURES WITH 6- AND 12-INCH.

Star	1900 Oct. 13	1902 Oct. 5	1900 Oct. 17	1902 May 14	Mean
n	.....	11.98	12.37	12.26	12.20
k	12.13	11.68	11.83	11.98	11.91

1900, July 20; 40-inch. Wedge II; seeing very fine.

Star	Scale Readings								Mean Scale Reading	Magnitude
	First				Second					
n	24.1	24.5	23.0	23.0	22.0	24.1	24.0	23.0	23.47	**12.20**
g	18.5	19.0	20.7	19.8	....	....	....	....	19.50	**11.22**
k	26.9	24.3	24.0	25.0	....	....	....	....	25.05	**11.91**
u	31.1	32.2	33.2	35.0	....	....	....	....	32.88	13.11
z	46.0	46.7	47.5	47.9	....	....	....	....	47.05	14.95
δ	57.1	52.0	52.2	51.2	54.5	....	....	....	53.40	15.69
η	36.4	34.0	35.1	34.0	....	....	....	....	34.88	13.37

Wedge constant, 0.130 mag.

1900, October 17, 40-inch. Wedge II; seeing poor.

n	32.5	32.9	34.0	31.4	28.1	30.9	31.63	**12.20**
k	28.0	30.0	30.9	31.0	30.0	30.9	30.13	**11.91**
η	41.6	41.8	40.5	39.7	39.6	40.8	41.30	13.32
δ	53.0	54.8	55.2	61.0	59.6	58.5	57.02	15.51
z	51.3	51.4	51.5	47.5	48.9	50.9	50.25	14.57
u	38.1	39.2	38.9	38.5	37.3	39.6	38.60	13.05
v	11.0	15.0	15.0	....	....	....	13.67±	9.8 ±

Wedge constant, 0.130 mag.

TABLE III—*Continued.*

RESULTING MAGNITUDES. FROM MEASURES WITH 40-INCH.

Star	1900 Oct. 17	1900 July 20	Mean
u	13.05	13.11	13.08
η	13.32	13.37	13.35
ε	14.57	14.95	14.76
δ	15.51	15.69	15.60

No correction for differing zenith distance has been applied to these measures, since even in case of the most distant comparison stars, *A*, *B*, and *C*, the distance would differ by less than half a degree, giving a correction of less than 0.01 mag. at the altitude at which the measures were made. For the fainter standards the correction would be still smaller.

MAGNITUDE CURVE.

Fig. I is platted with the photometric magnitudes as abscissæ and the positions in the light-scale as ordinates, giving the

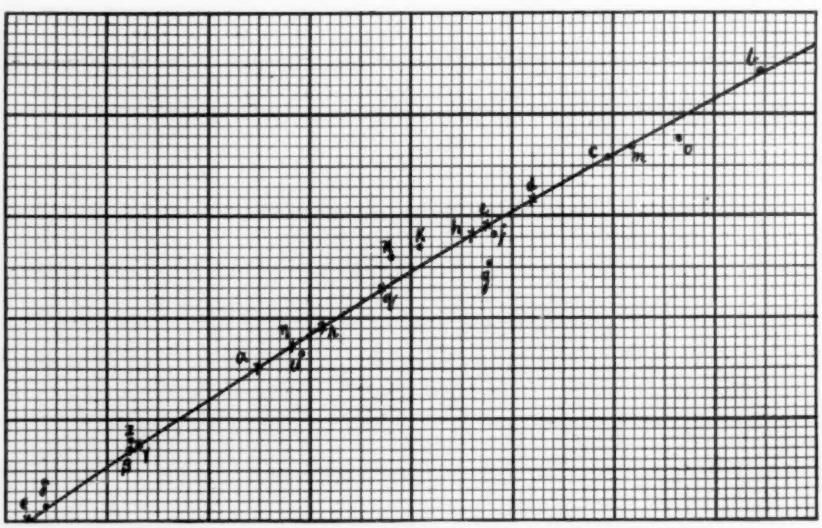


FIG. 1.

points indicated by the round dots. Through these the smooth curve was drawn and on this curve the stars not measured with the photometer were entered from their positions in the light scale, the places being indicated by crosses. The average distance between the platted points and the curve is 0.11 magnitude, the greatest residual being for the star *g*, 0.30. This depends on the measures of only two nights. Omitting this star, the mean residual is reduced to 0.09. The mean value of one step is 0.16 magnitude, and is essentially the same with the different apertures.

VISUAL OBSERVATIONS OF THE VARIABLE.

The visual comparisons by Argelander's method, and reductions to magnitudes, are given in Table IV, which should be printed as a double-page table, the comparisons on the left and the corresponding reductions on the right, For convenience the current number is repeated on the right-hand page. When the variable was invisible, the limit of vision was estimated from the faintest comparison star seen, and placed in the mean-step column, preceded by the inequality sign, which is read "fainter than" the quantity following. The limit of vision from these observations is 13.24 for the 6-inch, and 14.76 (perhaps, the record is "1900, April 4, 12-inch, power 275, *z* glimpsed) for the 12-inch.

THE LIGHT-CURVE.

In Fig. 2 the observed magnitudes are platted, showing the star's variations from 1896 to 1903. The magnitudes at maximum vary from 9.0 to 10.5, while only one minimum, that of June 4, 1900, is well enough observed to determine the limit, 15.4 magnitude. A single observation near the following minimum, July 9, 1901, at 15.0 magnitude, confirms the first, as far as it goes.

TABLE IV.

6871 V Lyrae.

Comparisons of the Variable by Argelander's Method.

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1896		C. S. T.	G. M. T.			
1	Sept.	22		3825	80	6	c 2 v, v 4 d
2		30	7	3833.5	40	6	c 1 v, v 4 d
3	Oct.	6	7	3839.5	40	6	c 2 v, v 3-4 d
4		13	7	3846.5	40	6	c 0-1 v, v 6 d, m 2 v
5		21	8	3854.6	40	6	v 2 c, m 1 v, o 3 v
6	Nov.	4	7	3868.5	40	6	m 4-5 v, c 2 v, v 4 d
7		12	7	3876.5	40	6	v d
					150	6	c 4 v, v 2 d, v 6 g
8		26	7	3890.5	150	6	d 1 v, v 5 e, v 1 h, v 2 g
					150	6	k 2 n, n 3 q, q 2 r
9	Dec.	9		3903	150	6	e 1 v, v g, v 3-4 n, h 1 v
10		23	6	3917.5	150	6	v 1-2 e, v g, v 3 n
	1897						
11	Jan.	9	6	3934.5	80	6	v not seen
12	May	25	9	4070.6	150	6	v not seen
13	July	1	9	4107.6	150	6	v not seen
14		26	9	4132.6	150	6	v not seen
15	Aug.	27	8	4164.6	150	6	q 2 v, v 0-1 r
16	Sept	16	7	4184.5	150	6	g 1 v, v e, v 6 k, k 3 q
17		20		4188	150	6	d 1-2 v, v 2-3 g, v 6-8 q
18		25		4193	40	6	v h, v 0-1 g, f 1 v
					150	6	v 2 g, v 1 f, d v
19	Oct.	1	7	4199.5	40	6	c 3-4 v, d 1 v, v 4 g
					150	6	c 4 v, d 1 v, v 4 g, v 3 f
20		7	7	4205.5	40	6	v f, d 1-2 v
					150	6	v 3 f, v 1-2 d, c 6 v
21		14	6	4212.5	150	6	v 4-5 f, v 2 d, c 4 v
22		23	7	4221.5	150	6	v 4 f, v d, c 5 v
23		29	6	4227.5	40	6	v 1-2 d, c 3 v
24	Nov.	3	7	4232.5	40	6	v 2 d, c 4 v
25		17	7	4246.5	150	6	c 4-5 v, v 1 d
26		20	6	4249.5	40, 150	6	d 4 v, v 1 g, v f
	1898						
27	June	25	10	4466.7	80	12	v not seen
28	July	7	10	4478.7		12	v not seen
29		20	10	4491.7		12	v not seen
30	Aug.	8	10	4510.7	80	12	v not seen
31		18	11	4520.8	80	12	v 1 u, q 10 v
					175	12	v u, r 2 v
32	Sept.	2	8	4535.6	150	6	v 3-4 f, v 3 e, d 3 v
33	Sept.	20	8	4553.6	40	6	v 1 c, m 0-1 v
34	Oct.	5	7	4568.5	40	6	v 2 m, v 4 c, b 6-7 v
35		28	7	4591.5	150	6	v 1-2 c, v m
36		31	7	4594.5	40	6	v 3 c, v 0-1 m
37	Nov.	7	6	4601.5	40	6	m 1 v, v c, v 8 e

TABLE IV.

6871 V Lyrae.

Reduction of Observations.

No.	Details in Steps	Means: Steps	Means: Mag.	Seeing	Remarks
1	33.9, 35.8	34.9	10.24	Moon, good	
2	34.9, 35.8	35.4	10.17	Moon, good	
3	33.9, 35.3	34.6	10.15	good	
4	34.4, 37.8, 35.0	35.7	10.10	Moon, fair	
5	37.9, 36.0, 34.6	36.2	10.02	good	
6	32.5, 33.9, 35.8	34.1	10.39	fair	
7	31.8 31.9, 33.8, 31.4	32.2	10.75	good, Moon	passing clouds
8	27.4 30.8, 34.2, 29.4	30.5	11.01	good	limit *r*
9	28.2, 25.4, 29.5, 27.4	27.6	11.52	fair, Moon	
10	30.7, 25.4, 29.0	25.8	11.81	good	
11		<27.0	<11.6	low, Moon	limit *k*
12		<19.0	<12.9	good	*k*, *n*, *q*, and *r* seen
13		<19.5	<12.9	good to fair	limit 3-4 < *q*
14		<19.5	<12.9	good to fair	limit 3-4 < *q*
15	21.0, 19.5	20.3	12.74	good	
16	24.4, 29.2, 33.0	28.8	11.32	fair	
17	30.3, 27.9, 30.0	29.1	11.29	good	
18	28.4, 25.9, 27.1 27.4, 29.1, 31.8	28.3	11.40	good	
19	32.4, 30.8, 29.4 31.9, 30.8, 29.4, 31.1	30.8	10.97	fair	
20	28.1, 29.3 31.1, 33.3, 29.9	30.3	11.07	Moon	
21	32.6, 32.8, 31.9	32.4	10.70	good	
22	32.1, 31.8, 30.9	31.6	10.83	good	
23	33.3, 32.9	33.1	10.57	good	
24	33.8, 31.9	33.4	10.52	good, Moon	
25	31.4, 32.8	32.1	10.76	fair to good	
26	27.8, 26.4, 28.1	27.4	11.55	good	
27		<15	<13.6		limit 8 < *q*
28		<16	<13.4	fine	limit *t*
29		<16	<13.4		limit 3 < *s* and *t*
30			<14	good	limit 2 mag. < *q*
31	17.4, 13.0 16.4, 17.0	16.1	13.38	good	
32	31.6, 32.2, 28.8	30.9	10.95	Moon, good	
33	36.9, 36.5	36.7	9.95	fair	
34	39.0, 39.9, 37.8	38.9	9.50	good	
35	37.4, 37.0	37.2	9.81	Moon, poor	
36	38.9, 37.5	38.2	9.62	Moon, good	
37	36.0, 35.9, 37.2	37.4	9.79	good	

TABLE IV — *Continued.*

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1898		C. S. T.	G. M. T.			
38	Nov.	12	6	4606.5	40	6	m v, v 1 c
39		19	6	4613.5	40	6	m 3 v, c 2 v, v 1 d
					150	6	c 4 v, v 1 d
40		30	6	4624.5	40	6	m 5 v, c 5 v, v d
					150	6	c 4 v, v 1 d
41	Dec.	7	7	4631.5	150	6	c 4 v, v d, v 3 e, v g, v 6 k
42		13	6	4637.5	150	6	d 5 v, v 1 e, v 4 g
43		30	6	4654.5	150	6	g 2-3 v, v 1-2 q
	1899						
44	Jan.	8	6	4663.5	150	6	e 4 v, v f, v 2 g
45		14	6	4669.5	150	6	f 2 v ±
46	Feb.	15	17	4702.0	200	6	q 4 v, v 1-2 r
47	Mar.	22	16	4736.9		6	v not seen
48	Apr.	16	15	4761.9	150	6	v or u suspected
49	May	1	9	4776.6	200	6	v not seen
50		29	9	4804.6	200	6	v not seen
51	June	7	9	4813.6	150	6	v not seen
52		24	9	4830.6	150	6	v not seen
53		26	9	4832.6	150	6	v not seen
54	July	6	10	4842.7	150	6	v not seen
55		11	9	4847.6	200	6	v not seen
56		29	9	4865.6	150	6	v not seen
57	Aug.	9	9	4876.6	150	6	r 1-2 v, v 1-2 u
58		12	9	4879.6	150	6	r 2 v, v 1 u
59		22	8	4889.6	150	6	q 2-3 v, v 0-1 r
60		30	9	4897.6	150	6	q 0-1 v, v 4 r
61	Sept.	4	8	4902.6		6	g 2 v, v 3 q
62		11	8	4909.6	150	6	v 2 g, d 2-3 v
63		26	8	4924.6	40	6	v 2 c, v 3-4 m, o 4 v
64	Oct.	6	7	4934.5	40	6	b 4 v, v 4 m, v 3-4 c, v 1 x, o 1 v
65		21	7	4949.5	40	6	b 1-2 v, o 1 v, v 5 m, v 4 x
66		28	7	4956.5	40	6	b 4-5 v, o 0-1 v, v 2 m
67	Nov.	4	6	4963.5	40	6	o 2-3 v, v 2 m
68		15	6	4974.5	40	6	v m, v 1 c, o 4-5 v
69		22	6	4981.5	40	6	o 5 v, m 3 v, c 1-2 v, v 3 d
70		27	7	4986.5	40	6	o 6-8 v, c 3-4 v, m 3 v, v d
					150	6	c 2-3 v, v 2-3 d
71		5	6	4994.5	40	6	c 3 v, v 2 d
					150	6	c v, v 3 d
72	Dec.	19	6	5008.5	150	6	d 2 v, v 4 e, v 2 g
	1900						
73	Jan.	1	6	5021.5	150	6	g 1 v, v 1 k, v 3 q
74	Feb.	22	17	5074.0	350	40	q 5-6 v, v 3 u
75	Mar.	22	15	5101.9	350	40	v u
76	Apr.	4	15	5114.9	275	12	v 4 u ±
77		6	16	5116.9	350	40	u 6 v, v 3-4 z, zx
78	May	2	15	5142.6	350	40	x 4 v, z 3 v, v 2 δ, v 4 ε
79		11	13	5151.8	350	40	z 5 v, v 3 δ
80		24	10	5164.7	275	12	v not seen

TABLE IV — *Continued.*

No.	Details in Steps	Means: Steps	Means: Mag.	Seeing	Remarks
38	37.0, 36.9	37.0	9.85	good	
39	34.0, 33.9, 32.8 33.9, 32.8	33.5	10.50	Moon, good	
40	32.0, 30.9, 31.8 31.9, 32.8	30.2	11.09	good	
41	31.9, 31.8, 32.2, 25.4, 33.0	30.9	10.96	good	
42	26.8, 30.2, 29.4	28.8	11.30	good	
43	22.9, 24.5	23.7	12.19	good	
44	25.2, 28.1, 27.4	26.9	11.62	good	
45		26	11.8	difficult	
46	19.0, 20.5	19.8	12.81	good	
47		<19	13.0		limit *r*
48		<23	12.3		limit *q*
49		<23	12.3	low, good	
50		<19	13.0	good, low	
51		<19	13.0	good, low	
52		<19	13.0	fair	
53		<19	13.0	good	
54		<19	13.0	good	
55		<16	13.4	good	
56		<19	13.0	good	
57	17.5, 17.9	17.7	13.15	good	
58	17.0, 17.4	17.2	13.03	good	limit *u*
59	20.5, 19.5	20.0	12.80	fair, Moon	limit *v*
60	22.5, 23.0	22.8	12.31	fair	limit *r*
61	23.4, 26.0	24.7	12.00		limit 1 < *u*
62	27.4, 28.3	27.9	11.48	Moon, good	
63	37.9, 40.5, 33.6	37.7	9.73	good	
64	40.3, 41.0, 39.4, 36.6, 40.1	39.5	9.37	good	
65	42.8, 36.6, 42.0, 43.1	41.1	9.07	good	
66	39.8, 37.1, 39.0	38.6	9.55	good	
67	35.1, 39.0, 40.9	38.3	9.61	good	
68	37.0, 36.9, 33.1	35.7	10.12	good	
69	32.6, 34.0, 34.4, 34.8	33.9	10.42	good	
70	30.6, 32.4, 34.0, 31.8 33.4, 34.3	32.8	10.61		
71	32.9, 33.8 35.9, 34.8	34.4	10.33	good	
72	29.8, 33.2, 27.4	30.1	11.08	good	
73	24.4, 28.0, 26.0	26.1	11.78	good	
74	17.5, 19.4	18.5	13.00	Moon, poor	
75		16.4	13.34	Moon, fair	
76		12.4	13.97	good	*z* glimpsed
77	10.4, 11.5	11.1	14.18	haze	
78	3.7, 5.0, 3.5, 4.0	4.1	15.18	good	
79	3.0, 4.5	3.8	15.21	fair, Moon	
80		<11	14.2	fair to good	limit 5 < *u*

TABLE IV—*Continued.*

No.	Date				Ocular	Aperture	Comparisions
	Month	Day	Hour	Julian Day			
	1900		C. S. T.	G. M. T.			
81	May	29	12	5169.8	237	40	*u* 5–6 *z*, *z* 5 *v*, *v* δ, *v* 4 ε
82	June	6	10	5177.7	350	40	*v* not seen
83		28	10	5199.7	237	40	*z* 4–5 *v*, *v* 5 ε, *v* 3 δ
84	July	20	10	5221.7	237	40	*z* 1 *v*, *x* 2–3 *v*, *v* 5 δ
85	Aug.	2	9	5234.6	460	40	*v* not seen
86		16	9	5248.6	350	40	*z* 1 *v*, *v* 1 *x*, *v* 4 ε
					350	40	*z* *v*, *v* 4 δ, *v* 6–8 ε
87		29	12	5261.8	460	40	*u* 6 *v*, α 4 *v*, *v* 4 *z*, *v* 3 *x*
88	Oct.	4	9	5297.6	237	40	*v* 5–6 *k*, *v* 3 *g*, *v* 3 *e*, *d* 1 *v*
89		10	7	5303.5	150	6	*v* 4 *e*, *c* 4 *v*
90		13	7	5306.5		6	
91		17	8			12	
92		26	6	5319.5	150	6	*v* 6 *k*, *v* 3 *g*, *v* 4 *e*, *c* 2 *v*
93	Nov.	13	6	5337.5	40	6	*m* 4–5 *v*, *c* 3 *v*, *v* 5 *f*, *v* 6 *g*
94		29	7	5353.5	40	6	*m* 5 *v*, *c* 5 *v*, *v* *d*, *v* 2 *g*
95	Dec.	11	6	5365.5	40	6	*m* 8 *v*, *d* 4 *v*, *v* 2 *g*
					150	6	*v* 1 *g*, *v* 8–10 *k*
96		29	8	5383.6	150	6	*g* 3 *v*, *v* 2–3 *k*
	1901						
97	July	9	9	5575.6	350	40	*z* 5 *v*, *v* *x*
98	Nov.	12		5701	350	40	*v* > *n* or *k*
	1902						
99	Mar.	5	14	5814.8	237	40	*u* 2 *v*, *v* α, *v* 6–7 *z*, *g* 4 *e*
100		28	15	5837.9	237	40	*z* 2–3 *v*
101	May	14	13	5884.8		12	*v* < *n* or *k*
102	Oct.	5	10	6028.7		12	
						12	*v* 4–5 *u*
103	Nov.	30	7	6084.5		24	
104	Dec.	26	7	6110.5	237	40	*v* d ±
	1903						
105	Mar.	27	16	6201.9	237	40	*n* 6–8 *v*, *v* 5 η, *v* 3–4 *u*
106	Apr.	3	16	6208.9	237	40	*u* 1–2 *v*

In Chandler's *Third Catalogue* the provisional elements of maximum from observations of 1893–1895, are given as—

Maximum = 1893 August 24 (J. D. 2412700) + 377 *E*.

These elements fairly represent the above maxima, but the agreement is somewhat improved by placing the zero epoch thirteen days earlier, giving the elements—

Maximum = 1893 August 11 (J. D. 2412687) + 377 *E*.

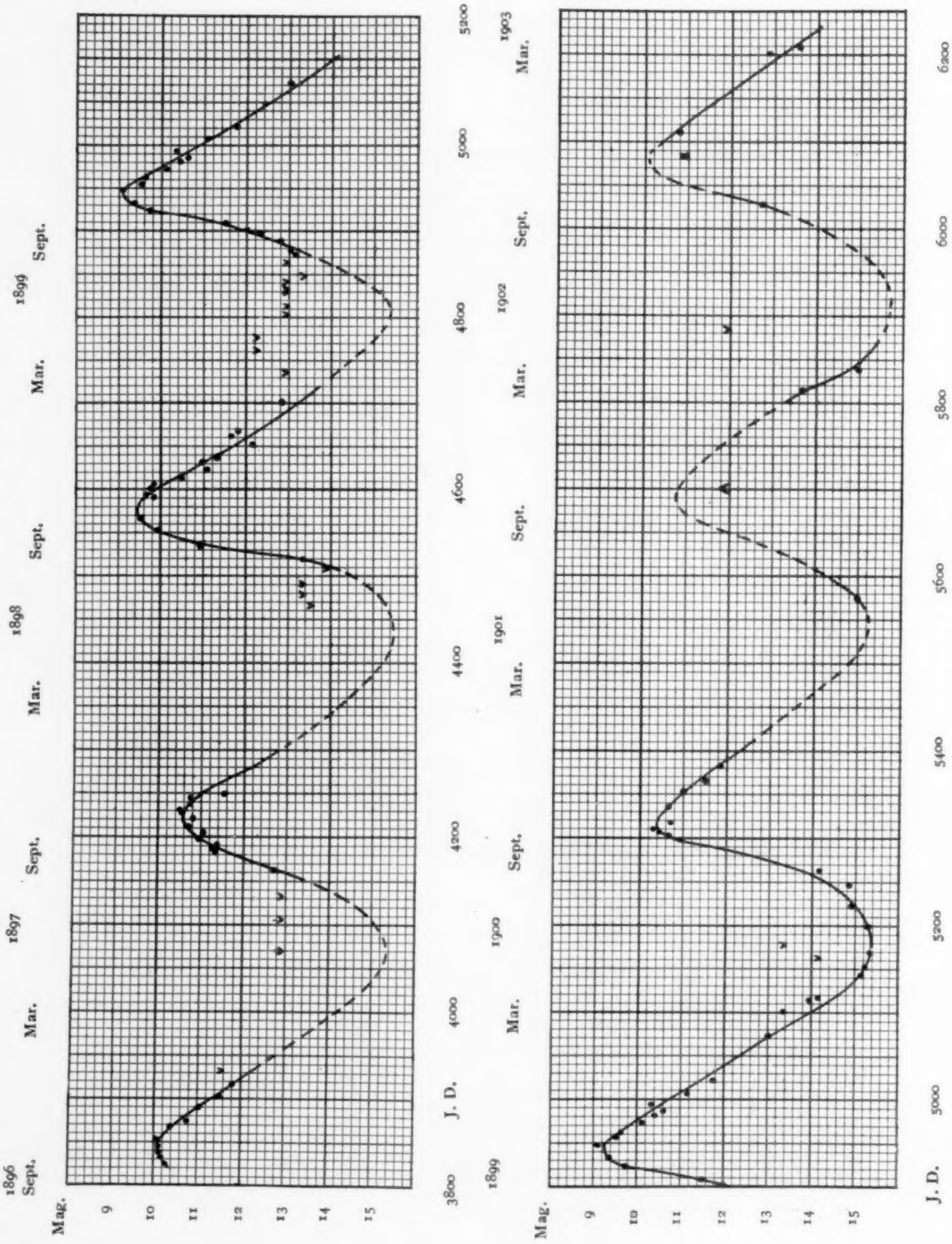
The agreement with these elements is shown by the following table:

TABLE IV — *Continued*

	Details in Steps	Means: Steps	Means: Mag.	Seeing	Remarks
81	3.0, 1.5, 4.0	2.8	15.37	fair to good	
82		<16	13.4	haze	limit *u*
83	3.5, 4.0, 4.5	4.0	15.20	good	
84	7.0, 5.2, 6.5	6.2	14.91	fine	
85					
86	7.0, 8.7, 4.0 8.0, 5.5, 7.0	6.7	14.86	fair better	
87	10.4, 11.3, 12.0, 10.7	11.1	14.17	good	
88	32.5, 28.4, 32.2, 30.8	31.0	10.94		*x* and *z* seen
89	33.2, 31.9	32.6	10.68	good	
90			10.45	good	photom., 6-in.
91			10.31	good	photom., 12-in.
92	33.0, 28.4, 33.2, 33.9	32.1	10.75	good	
93	32.5, 32.9, 33.1, 31.4	32.5	10.69	good	
94	32.0, 30.9, 31.8, 27.4	30.5	11.00	Moon	
95	29.0, 27.9, 27.4 26.4 (36.0)	27.7	11.52	good	
96	22.4, 29.0	25.7	11.83	Moon	
97	3.0, 7.7	5.4	15.00	fair	*v* & *x* near limit
98		<26	<11.8		
99	14.4, 14.5	14.5	13.68	fair	
100		5.5	15.00	Moon	
101					*v* is < *n* or *k*
102		20.9	<12 12.76		photometer
103			10.9		photograph
104		31.8	10.08	good	
105	19.0, 19.9, 19.9	19.6	12.87	good	
106		14.9	13.6	clouds	

Maxima: Epoch	Cal. J. D.	Obs. J. D.	Obs. Cal.	O-C.	Minima: Epoch	Cal. J. D.	Obs. J. D.	Obs. Cal.	O-C.
3	3818	3845	1896, Oct. 12	−27	3	3677	....	....	....
4	4195	4220	1897, Oct. 22	−25	4	4044	4070	1897, May 25	−26
5	4572	4575	1898, Oct. 12	− 3	5	4421	4433	1898, May 23	−12
6	4949	4940	1899, Oct. 12	− 9	6	4798	4808	1899, June 2	−10
7	5326	5315	1900, Oct. 22	− 9	7	5175	5175	1900, June 4	0
8	5703	....	....	....	8	5552	5545	1901, June 9	− 7
9	6080	6080	1902, Nov. 26	0	9	5929	....	....	....

FIG. 2.



LIGHT CURVE OF *V LYRAE*.

The large residuals of epochs 3 and 4 require mention. The observations began too late to fix maximum 3 with precision. It doubtless took place about twenty days earlier; in fact, the calculated date, J. D. 3818, is the mean of the dates given by Yendell and H. M. Parkhurst (Max. A). For epoch 4 the curve at maximum is not of the usual shape, the difference accounting for something like twenty days, and in the right direction to improve the agreement with the ephemeris. The residuals for the remaining maxima are within the errors of observations.

I wish to acknowledge the assistance rendered, as in the work on *X Cephei*, to Miss Bloodgood, and Messrs. Ellerman and Sullivan.

Yerkes Observatory,
April 1903

PECULIARITIES AND CHANGES OF FRAUNHOFER LINES INTERPRETED AS CONSEQUENCES OF ANOMALOUS DISPERSION OF SUNLIGHT IN THE CORONA.[1]

By W. H. JULIUS.

ATTENTION has been drawn to several variable peculiarities of Fraunhofer lines, mainly by Jewell's investigations on the coincidence of solar and metallic lines.[2] Here we do not mean the irregularities occurring in the spectrum of spots or of faculæ, which relate to disturbances in comparatively small parts of the Sun, but abnormalities shown by average sunlight, as observed when the slit is illuminated by a long strip of an imperfectly focused solar image. In that case, according to Doppler's principle, we may, of course, expect displacements of the lines in consequence of the Sun's rotation, the rotation of the Earth, and the change in the distance between Sun and Earth caused by the eccentricity of the Earth's orbit. But even when all these influences have been allowed for, some irregularities still remain.

Indeed, Jewell has observed that while some Fraunhofer lines exactly coincide with the emission lines in the arc spectrum of elements, others do not, and that the displacements are unequal both for lines of different elements and for the various lines of one and the same element. Moreover, the shifting of certain lines on one set of photographic plates was sometimes found to be different from that on a set of plates taken at another time. With several lines the intensity also appeared to be variable.

Jewell explains these phenomena on certain hypotheses as to

[1] Communicated to the Royal Academy of Sciences, Amsterdam, at the meeting of February 28, 1903.

[2] L. E. JEWELL, "The Coincidence of Solar and Metallic Lines: A Study of the Appearance of Lines in the Spectra of the Electric Arc and the Sun," ASTROPHYSICAL JOURNAL, 3, 89–113, 1896; "Spectroscopic Notes: Absolute Wave-Lengths, Spectroscopic Determinations of Motions in the Line of Sight, and Other Related Subjects," *ibid.*, 11, 234–240, 1900.

density, pressure, and temperature of the absorbing and emitting gases in the different layers of the solar atmosphere, and by variable ascending and descending velocities of matter.

HALE'S ABNORMAL SOLAR SPECTRUM.

Much greater irregularities than those mentioned are found in an "abnormal" solar spectrum, lately described by G. E. Hale.[1]

This highly remarkable spectrum was accidentally photographed as long ago as February, 1894, in a series of exposures, made with the sole intention of investigating the peculiarities of the grating. Only a few months later it was discovered that a very extraordinary phenomenon had been photographed. Hale hesitated to publish this accidental discovery. Copies of the plate were sent to several spectroscopists for examination, with the request that an explanation referring the phenomenon to some origin other than solar be supplied, if possible. As no such explanation was forthcoming, the spectra were very carefully measured and described.

On one and the same plate twelve exposures had been successfully made in the third-order spectrum of a plane grating. A solar image 51 mm. in diameter was so adjusted that the image of a spot fell exactly on the slit. The length of the slit (6.5 mm) corresponded to about one-eighth of the Sun's diameter.

The first exposures show the normal spectrum without any considerable changes. Then came the disturbance which culminated in the eighth spectrum, and in the following four decreased rapidly. Hale gives reproductions of four spectra, each of them extending from λ 3812 to λ 4132. No. 1 was taken before the disturbance occurred; No. 2 is the most abnormal spectrum; No. 3 is called by Hale the "intermediate" spectrum; it was obtained a few moments after the abnormal one; No. 4 shows once more the normal solar spectrum, as it was photographed at another time on another plate. Nos. 1, 2, and 3

[1] "Solar Research at the Yerkes Observatory," ASTROPHYSICAL JOURNAL, **16**, 211–233, 1902.

show a dark band throughout the whole spectrum, corresponding to the Sun-spot which had been focused on the slit.

The most prominent features of the abnormal spectrum are:

1. The band due to the spot appears much fainter than in the spectra photographed before and after the disturbance.

2. In the case of several Fraunhofer lines the intensity or the width is *greatly diminished*. This is most conspicuous with the broad, dark calcium bands H and K and with the hydrogen line $H\delta$, these being almost totally absent in the abnormal spectrum.

3. Other Fraunhofer lines, on the contrary, appear *uncommonly strengthened*.

4. Many lines are more or less displaced.

The same peculiarities are noticed, though generally in a lesser degree, in the intermediate spectrum, so that the latter, in fact, forms a link between the abnormal and the normal spectrum.

This marvelously complicated disturbance was not confined to light coming from a comparatively small part of the solar disk, for instance from the immediate surroundings of a spot; on the contrary, it extended almost equally over the whole width of the spectrum and was therefore nearly the same for all the light which came from a very great area of the Sun.

The moments of the twelve exposures and the exact date had not been recorded; but there was sufficient evidence that the whole process of the disturbance lasted only a very short time. Hale calls the phenomenon "a remarkable disturbance of the reversing layer." But is it not almost impossible to imagine a rather thin layer in the solar atmosphere undergoing suddenly and simultaneously over a great part of the Sun a change so great as to make its absorbing and radiating power in some parts of the spectrum for a while nearly unrecognizable?

It occurred to me, therefore, that the origin of the phenomenon should be looked for somewhere on the path of the light between the Sun and the Earth. If on this path there be media causing anomalous dispersion, the beam must show an altered composition.

As I formerly indicated,[1] the properties of the chromospheric light may be derived from the supposition that this light has been separated from the photospheric light by anomalous dispersion. According to this hypothesis the spectrum of the chromosphere informs us which are the kinds of light that may follow rather strongly curved paths in the solar atmosphere. So the idea suggested itself that the same waves might play a striking part in Hale's abnormal spectrum.

In order to investigate the question as impartially as possible, I marked (before consulting Hale's table or a table of chromospheric lines) on the reproductions of the spectra in the ASTROPHYSICAL JOURNAL a number of lines which struck me as being *weakened* in the abnormal spectrum. By means of George Higgs' photographic atlas of the normal solar spectrum the wave-lengths of the selected lines were easily read. They are to be found in the first column of Table I.

The second, third, and fourth columns show the intensities of these lines in the normal, the intermediate, and the abnormal spectrum as given by Hale (for the normal spectrum from Rowland's tables, for the other two from estimates by Mr. Adams). Hale remarks that the intensities of the lines were estimated independently for the two disturbed spectra.[2] The fifth column indicates the intensities of corresponding chromospheric lines as found by Lockyer in the spectrum secured at Visiadrug[3] during the 1898 eclipse; the sixth column shows the absorbing substances.

Table II has been prepared in a similar way; here we find the lines, which on the reproduction appeared to be *strengthened* in the abnormal spectrum.

[1] ASTROPHYSICAL JOURNAL, **12**, 185–200; **15**, 28–37; *Physikalische Zeitschrift*, **4**, 85–90; 132–136.

[2] In selecting the lines that appeared weakened in the abnormal spectrum, I of course compared the three spectra together. That is why in my table some lines occur whose intensities as estimated by Mr. Adams are not comparatively low in the abnormal spectrum.

[3] LOCKYER, CHRISHOLM-BATTEN, AND PEDLER, "Total Eclipse of the Sun, January 22, 1898.—Observations at Visiadrug," *Phil. Trans.*, A, **197**, 151–227, 1901.

TABLE I.

LINES WHOSE INTENSITY IS LESS IN THE ABNORMAL THAN IN THE NORMAL SPECTRUM.

Wave-Length	Intensity				Elements	Remarks
	Normal (Rowland)	Intermediate (Hale)	Abnormal (Hale)	Chromosphere (Lockyer)		
3871.4				4	*C*	Not mentioned in Hale's list, but distinctly weakened in the abnormal spectrum on reproduction.
3872.6				4	*Fe*	
3874.09	4	9	..	2(?)	*Fe*	
3878.47	22	25	..	3–3	*Fe, Fe*	$\lambda = 3878.15$ and $\lambda = 3878.72$. Hale mentions *Fe, Mn*.
$H\zeta$ 3889.05	?	15	..	8	*H*	
3895.80	7	12	..	3	*Fe*	
3899.30	5	4	..	2	*V?*	
3903.09	10	12	..	2–3	*Fe*	
3905.66	12	20	..	2	*Cr, Si*	
3906.70	14	..	4	2	*Fe*	
3913.63	9	7	..	6	*Ti*	
3914.49	7	8	5*		*Ti*	* These intensities are very probably estimated too high when compared with the numbers in the second column. *Cf.* note 2 on p. 53.
3916.54	3	..	4*	3	*V*	
3920.41	10	10*	10*	3	*Fe*	
3923.05	12	12*	12*	3	*Fe*	
K 3933.82				10	*Ca*	
3944.16	15	15*	12*	5	*Al*	
3948.91	13	15	..	3	*Fe*	
3950.10	5	..	2	3	*Fe*	
3953.02	17	15	..		*Fe*, etc.	
3958.35	5	8	..	4	*Ti*	
3961.67	20	20	..	6	*Al*	
H 3968.63	(700)	7	7	10	*Ca*	
$H\epsilon$ 3970.18	7	8	..	10	*H*	
3977.89	6	8	..	2	*Fe*	
3986.90	6	8	..		....	
3998.78	4	4	4*	4	*Ti*	
4012.50	5	4	5*	5–6	*Ti*, etc.	
4033.22	7	12	3	3–4	*Mn, Fe*	
4034.64	6	10	..	3–4	*Mn, Fe*	
4045.98	30	30	5	7	*Fe*	
4063.76	20	20	..	6–7	*Fe*	
4071.91	15	15	15*	6	*Fe*	
4077.88	8	10	7*	10	*Sr*	
$H\delta$ 4102.00	40	7	..	10	*H*	

The result is very striking. *Weakened lines correspond to chromospheric lines almost without exception; most of the strengthened lines, on the other hand, are not to be found in the spectrum of the chromosphere.*

Lockyer gives the strength of the chromospheric lines on a scale such that 10 indicates the strongest and 1 the faintest lines. If we take into account that in his list the greater part of

TABLE II.

LINES WHOSE INTENSITY IS GREATER IN THE ABNORMAL THAN IN THE NORMAL SPECTRUM.

Wave-Length	Intensity				Elements	Remarks
	Normal (Rowland)	Intermediate (Hale)	Abnormal (Hale)	Chromosphere (Lockyer)		
3921.86	4	..	20		*Zr, Mn*	
3927.77	..	..	25		?	
3930.45	8	15	28	3-4	*Fe*	
3937.39	..	..	10		?	
3940.25	..	7	12		?	
3950.50	2	..	13		*Y*	
3962.29	3	..	11		*Fe?*	
3973.77	6	..	15	2 (?)	*NiZrFeCa*	
3981.92	4	13	30	6*	*Ti, Fe*	* In Humphreys' table of chromospheric lines (1901 eclipse) this line does *not* occur.
3992.97	3	4	10		*V, Cr*	
3996.80	..	..	9		?	
4013.90	8	12	15		*Ti, Fe*	
4014.67	5	9	20		*Fe*	
4023.38	..	..	10		?	
4033.77	2	3	15		*Mn*	
4040.79	3	6	20	4	*Fe*	
4044.09	5	20	15		*Fe*	

the lines bear the numbers 1 and 2, our table shows us that by merely observing the abnormal solar spectrum we have been able to pick out *strong* chromospheric lines. This cannot be chance. Undoubtedly both phenomena—the weakening of Fraunhofer lines in the abnormal spectrum and the origin of the chromospheric spectrum—are to be explained in close relationship with each other.

The *strengthening* of lines in the abnormal spectrum does not, on the contrary, seem to be so directly connected with the composition of the chromospheric spectrum.

If our view be correct that the chromospheric light has been separated by strong ray-curving from the "white" light emitted by deeper layers, those special radiations must, as a rule, show reduced intensity in the spectrum of the Sun's disk.[1] Fraun-

[1] It might be thought that the rays forming the chromospheric light need to be absent only from the spectrum of the *edge*, but not from that of the central portions of the Sun's disk. By a simple consideration following from a glance at Fig. 4 of my paper read in February, 1900 (ASTROPHYSICAL JOURNAL, **12**, 191), we see, however, that the chromospheric light visible to us may very well, in part, have its origin even

hofer lines corresponding to chromospheric lines will therefore have a more or less darkened background in the ordinary solar spectrum. The rate of darkening at various distances from the center of an absorption line is, of course, connected with the shape of the dispersion curve near that line; whereas the average shading depends also (1) on the quantity of matter causing anomalous dispersion and (2) on the slopes and directions of the density gradients in the gases through which the light is transmitted, viz., on the Sun's "activity."[1]

We distinguish, therefore, a twofold origin of the dark lines in the solar spectrum, viz.: real *absorption* of those waves which exactly correspond to the periods of the media and *dispersion* of the strongly deviated[2] neighboring light.

The dispersion will be especially evident where extraordinary differences in the density of the medium occur; in this way the widening of most of the Fraunhofer lines in the spectra of spots may be accounted for.

Dispersed light has not, of course, vanished; the absence of certain rays in the spectrum of a spot is counterbalanced by the increased intensity of the same radiations in the light coming from the neighboring faculæ. Thus the distribution of the density in the solar gases may locally be such that a limited part of the disk seems to emit a considerable amount of rays with abnormally high or abnormally low refractive indices. In the spectrum of this part not only will the Fraunhofer lines appear

in points of the Sun which lie opposite to the Earth's direction. The chromospheric light reaching the Earth may proceed from *any* point of Schmidt's "critical sphere." For the greater part it is likely to come from the back half of the Sun. But then the half facing us furnishes the chromospheric light which travels to other regions of the universe, and this light, of course, is wanting in the spectrum of the disk. There is some reason for supposing that, on an average, more chromospheric light is sent forth in directions making great angles with the Sun's equator than to the equatorial regions, including the Earth's orbit.

[1] The possible influence of the general or regular ray-curving (on Schmidt's principle) on the appearance of the spectral lines has in the present paper been left out of consideration. If we were able to observe or to calculate the radii of the "critical spheres" for radiations undergoing anomalous refraction, it would be possible to estimate that influence; but as yet sufficient data are wanting.

[2] ASTROPHYSICAL JOURNAL, **12**, 191, 1900.

narrower and fainter than usually, but we may even meet with lines contrasting ***brightly*** with their surroundings. These bright lines will not coincide with the corresponding absorption lines; their average wave-length will in general be greater or smaller than that of the absorbed light, for, according to the accidental distribution of the density, we shall find either the rays with high or those with low refractive indices most prominent in the beam.

The above considerations suggest an explanation of Hale's abnormal spectrum. In fact, the lines showing especially faint in this spectrum were exactly those which cause strong anomalous dispersion—witness the chromospheric spectrum. With H, K, $H\delta$, and some iron lines it is very evident that the abnormal faintness refers mainly to the broad dark shadings of the lines, *i. e.*, those parts whose darkness in the normal spectrum we attributed, not to absorption, but to dispersion. Moreover, the dark band due to the spot has nearly disappeared. This means that waves, which in normal circumstances are wanting in the spot spectrum on account of their strong dispersion, at the time of the disturbance had been gathered again into the beam, reaching the instrument. How all this may happen will become evident as soon as we shall be able to establish a plausible cause by which, within an angular space great enough to include a considerable part of the solar disk, the ***strongly dispersed rays might be gathered again.***

It is not necessary to introduce a new hypothesis for the purpose. The same idea about the Sun's constitution which enabled us to explain the properties of the chromosphere and the prominences[1] furnishes us once more with the required data. Indeed, if (according to Schmidt's theory) the Sun is an unlimited mass of gas, surfaces of discontinuity must exist similar to those whose general feature has been determined by Emden[2] for a sharply outlined radiating and rotating Sun. These surfaces must extend to the remotest parts of the gaseous body—a conclusion in excellent harmony with the visible structure of the corona. For along the surfaces of discontinuity waves and whirls are formed; the core-lines of the vortices nearly coincide

[1] *Physikalische Zeitschrift*, **4**, 85–90. [2] ASTROPHYSICAL JOURNAL, **15**, 38–59, 1901.

with the generatrices of the surfaces of revolution, and in these cores the density is a minimum. This may account for the streaky appearance shown more or less distinctly in all good photographs and drawings of the corona.

This particular appearance may have another cause, however, but for what follows this is immaterial. We only assume that the density of the coronal matter varies in such a way as to correspond to the striped structure visible at the time of a total eclipse of the Sun.

A coronal streamer which at a given moment runs exactly in the direction of the Earth may be very roughly compared, then, to a bundle of glass tubes through which we are looking lengthwise. Such a structure will gather and conduct rays of various directions, entering it at one end. This takes place also if the parts with the greater and those with the smaller optical density do not alternate abruptly, but gradually.

In Fig. 1 the optical density of the matter may be represented by the compactness of the streaking. A ray for which the medium has a large positive refraction constant would, for instance, follow the path AA', curving round the denser parts of the structure; a ray BB', for which the medium possesses a large negative refraction constant, would move in a similar way through the more rarefied regions. On the other hand, the light CC' for which the constant exactly equals zero is not influenced by the fluctuations of the density; and if for some kind of light the refraction constant is very nearly zero, the ray would have to travel a long way almost parallel to the structure before its curving would be perceptible.

Now the corona sometimes shows exceedingly long, pointed streamers. We only have to suppose *that the Earth was exactly in the direction of such a streamer at the moment the abnormal spectrum was photographed;* then all the irregularities observed in this spectrum become clear. Light under normal circumstances absent from the solar spectrum through strong dispersion has been collected by the coronal streamer; hence the *weakening* of the Fraunhofer lines, especially, also, of those in the spectrum of the spot. As the abnormalities were caused by a peculiar

distribution of matter in the vast regions of the corona lying between the source of light and the Earth (and not by disturbances in a relatively thin "reversing layer"), they could appear in the same way over a great part of the Sun's disk. The *rarity* of the phenomenon is the result of the slight chance we have to take a photograph at the very moment on which an uncommonly long coronal streamer is projected exactly on the part of the Sun's disk illuminating the slit; finally, the *short duration* is a consequence of the Earth's orbital motion, and probably of the rotation of the corona.

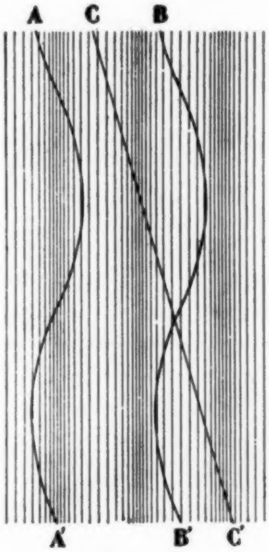


FIG. 1.

As we have mentioned before, *no* chromospheric lines correspond, in general, to those lines appearing extraordinarily *strong* in the abnormal spectrum. How are we to account for the strengthening of these lines?

We might be tempted to think of absorption in the corona; for if it be true that a streamer was turned toward the Earth, the rays had to go an uncommonly long way through an absorbing medium. But on closer examination this idea is less probable.

The particles of the extremely rarefied coronal gases will hardly influence each other; their periods will, therefore, be almost absolutely constant, so as to cause very sharp, narrow absorption lines. Thus it is difficult to understand how an absorption line already present in the normal solar spectrum might be strengthened by the absorbing power of the corona. Further, in studying Hale's table, we observe that many lines which are strong in the abnormal spectrum show a much smaller intensity in the intermediate spectrum (taken only a few moments later); while the reverse happens as well, viz., that lines are strong in the intermediate and very weak in the abnormal spectrum. This hardly fits in with the absorption hypothesis. Some lines showing this peculiarity are given in Table III:

TABLE III.

LINES WHOSE INTENSITY IS VERY DIFFERENT IN THE INTERMEDIATE AND THE ABNORMAL SPECTRUM.

Wave-Length	Intensity				Elements	Remarks
	Normal (Rowland)	Intermediate (Hale)	Abnormal (Hale)	Chromosphere (Lockyer)		
3905.66	12	20	..	2	*Cr, Si*	
3905.81	21	..	20		*Si*	
3921.71	9	14	..		*Ti, La, Zr, Mn*	
3921.87	4	..	20		*Zr, Mn*	
3950 33	..	10	..		?	
3950.51	2	..	13		*Y*	
3972.30	2	12	..		*Ni*	
3972.61	2	..	12		?	
4005.86	3	25	5		?	
4057.39	4	..	15	1–2	*Co, Fe*	
4957.66	7	10	..		?	

In the chromospheric spectrum corresponding lines seem to be wanting (at λ 3905.66 and λ 4057.39 the faint chromospheric line may possibly belong to another element than the abnormally strengthened absorption line).

To arrive at a more satisfactory explanation of the strengthening phenomenon, we suppose that these absorption lines do indeed cause anomalous dispersion of neighboring waves, but in a very slight degree. Then, the refractive indices of the neighboring waves differing but little from unity, the direction of those rays will be perceptibly changed only after they have traveled a very long way through the corona and almost parallel to its structure lines. Whereas the strongly refracted rays, entering the coronal streamer in various directions, were obliged to follow the structure lines, curving about them, and so in a sense were concentrated on the Earth, it may happen with the very slightly curved rays we are now considering that they have been bent, for instance, only once over the whole length of the streamer and continue their way in a direction not meeting the observing station. The divergence of a beam consisting of these rays will have increased, the intensity diminished. Thus the resultant spreading of neighboring light causes the absorption

line to appear somewhat widened, and therefore strengthened. But obviously it must be possible, too, that after a short time, under the influence of another part of the corona, circumstances assist that slightly curved light to reach the observer. In that case the absorption line is weak again. (Similar alternations, of course, also occur with the more strongly refracted rays, and that in quicker succession; but this does not alter the fact of their *average* intensity appearing increased as long as the structure lines of the coronal streamer are turned toward the spectroscope. For a detailed discussion of this case see the note at the end of this paper.)

In both abnormal spectra a number of absorption lines are more or less displaced. Perhaps this is partly due to motion in the line of sight, but after the foregoing it will not be necessary to explain in detail that anomalous dispersion also can account for this phenomenon. Dissymmetric form of the dispersion curve as well as a peculiar distribution of the density of the coronal matter may unequally affect the intensity of the light on both sides of the absorption line, and thus bring about a seeming displacement of the line.

CERTAIN PECULIARITIES OF LINES IN THE NORMAL SOLAR SPECTRUM.

If we have been right in connecting the uncommonly great abnormalities in Hale's spectrum with a very particular position of the Earth with respect to the corona, it is to be expected that similar irregularities, though in less degree, will continually be found, as the sunlight always reaches us through the corona.

According to Jewell's above-mentioned investigations, this supposition proves to be well founded. Many solar lines have varying intensities and positions, so that Jewell deems them unfit for standards for very accurate determinations of wave-lengths. And these are for the greater part the most prominent lines of the spectrum, especially the shaded ones.[1]

Jewell emphasizes the fact that all distinctly shaded lines in the solar spectrum show to a greater or less degree the following

[1] ASTROPHYSICAL JOURNAL, 11, 236, 1900.

typical feature:[1] With a broad, shaded, moderately dark background a much darker central absorption line contrasts rather sharply (Fig. 2). Besides, the absorption curve often shows depressions close to the central line, as in Fig. 3, sometimes symmetrical, sometimes dissymmetrical. Jewell affirms that this is not an optical delusion, due to contrast, but a real phenomenon. He assumes, therefore, that the broad absorption band is produced in the lower portions of the solar atmosphere and under a great range of pressure; that in higher levels radiation prevails again, producing a rather wide emission line; and that finally in the highest parts, where the pressure is very much less, the sharp absorption line is produced. The position of this central absorption line with respect to the emission line is usually unsymmetrical, which is conspicuous in the case of H and K. The central line itself also varies somewhat in width upon different plates, and its maximum of intensity is not always in the middle of the line. The displacement of this central line in H and K varies in magnitude, but, so far as has been observed, always toward the red with respect to the emission line and the corresponding metallic line (in the arc). Jewell concludes that the absorbing calcium vapor descends all over the solar surface with a velocity sometimes amounting to about seventy-five miles per minute.

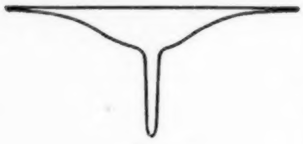


Fig. 2

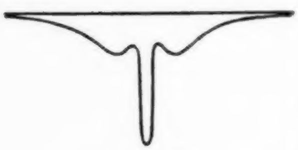


Fig. 3.

Upon the same plates showing strong dissymmetry in H and K the shaded lines of other elements (*Fe*, *Al*, *Mg*, *Si*) have been examined. The strongest iron lines and one aluminum line showed displacements of the same character as that observed in the case of H and K, but to a much smaller degree, and sometimes toward the violet, sometimes toward the red. Certain

[1] "Certain Peculiarities in the Appearance of Lines in the Solar Spectrum and Their Interpretation," *Ibid.*, 3, 1896.

shaded lines of *Mg* and *Si*, on the contrary, showed no evidence of a displacement, nor did the iron lines without considerable shading, the faint calcium line at λ 3949,056, and many other lines.

If we admit no other explanation of line-shifting and widening besides those based on Doppler's principle and on the effect of pressure and temperature, we arrive at very strange conclusions with regard to the condition of the elements in the solar atmosphere. Not less surprising, as noticed by Jewell,[1] is the small amount of the absorption in the shaded parts of the lines, when we consider the enormous depth of the solar atmosphere and the high pressure which must exist in the absorbing layers for them to produce a broad absorption band.

By making various suppositions concerning the condition of the gases in the solar atmosphere, Jewell succeeds in finding an interpretation of most of these astonishing facts. But it must be granted that his explanations include a greater number of arbitrary and mutually independent hypotheses than is the case with our explanations, founded as they are on selective ray-curving and readily deduced from that principle for each separate phenomenon, without introducing new suppositions.

Only the dark central lines of the Fraunhofer lines are to be ascribed, in our theory, to real absorption. Their shaded background of varying intensity we consider as an effect of anomalous dispersion of the not absorbed neighboring waves. This selective scattering will be strongest in those places where the density-gradients are relatively steep, viz., in whirls in the deeper regions of the gaseous body. But some of the widely dispersed rays may be gathered by the corona owing to its "tubular" structure, and be conducted along its greater or smaller streamers. This will especially apply to the most strongly refracted waves, whose position in the spectrum is very close to the real absorption lines ; thus pseudo-emission lines are produced in about the middle of the pseudo-absorption bands.[2]

[1] ASTROPHYSICAL JOURNAL, 3, 106, 1896.

[2] A most remarkable fact is that the shading of H, K, the iron line λ 3720.086 and of some other strong shaded lines is sometimes partially broken up in a series of faint

Probably Hale's abnormal spectrum has shown us a case where these seeming emission bands acquired an uncommon extent. We may therefore expect that a systematical investigation of solar spectra, photographed at different times, will afford all kinds of intermediate cases.

It would be desirable, for the moments when the photographs are taken, to know the form and position of the coronal streamers directed toward the Earth. At all events the actual phase of the Sun-spot period, with which the shape of the corona seems to be connected, should be taken into consideration ; and perhaps the simultaneous observation of the photospheric reticulation discovered by Janssen may procure some evidence concerning the position of coronal streamers, and thus contribute to our knowledge of their influence on the Fraunhofer spectrum.

UTRECHT, April 7, 1903.

nebulous lines, symmetrically situated about the central line (JEWELL, ASTROPHYSICAL Journal, **3**, 108, 1896). It might have been predicted by our theory that we should meet with this phenomenon. Let us consider a beam of light of an exactly defined wave-length belonging to the shaded background of an absorption line. This beam leaves the deeper layers of the Sun with a certain divergence. As it passes along a "tube" of the corona, its divergence will alternately diminish and increase, and on reaching the Earth it shows in the spectrum an intensity depending on the divergence (or, perhaps, convergence) with which it has left the last traces of the corona. For a beam of light whose wave-length is only slightly nearer to that of the absorption line, the medium will have a considerably greater refraction-constant, so that the rays of this beam on their way through the corona may make part of a bend more than the former ones. The beam may therefore arrive with a quite different degree of divergence, and, consequently, of intensity. Thus, proceeding toward the absorption line from either side, we easily see that we must meet with a periodically changing intensity. Rays corresponding to the middle of one of the fringes so formed will have made one full bend more or less than the rays belonging to the middle of the next fringes.

If this interpretation be correct, the width and the number of fringes visible must prove to be variable. So far as I know, the observations made on this point are not numerous. I trust that the proposed views may serve to further the investigation of this interesting phenomenon.

THE WAVE-LENGTHS OF THE SILICON LINES $\lambda 4128$ AND $\lambda 4131$ AND OF THE CARBON LINE $\lambda 4267$.

By J. HARTMANN.

SUPPLEMENTING my recent determination[1] of the wave-length of the magnesium line $\lambda 4481$, I should like to communicate my measurements on three other lines which, like the magnesium line, occur as well-measurable lines in stellar spectra of the first type, although hitherto their very diffuse character has not permitted an accurate laboratory determination of their wave lengths.

The lines at $\lambda 4128$ and $\lambda 4131$ are among the strongest in the spark spectrum of silicon. I will here mention only the following of the earlier determinations of their wave-lengths:

Eder and Valenta (*Denkschriften d. Wiener Akad.*,[2] **60**, 260, 1893)	4128.5	4131.5
Lockyer (*Proc. R. S.*, **61**, 443, 1897)	4128.6	4131.4
Exner and Haschek (ASTROPHYSICAL JOURNAL, **12**, 49, 1900)	4128.1	4131.0

It may readily be seen from the great difference in the above values how uncertain the wave-lengths of these lines have been hitherto on account of their diffuseness. Like the magnesium line, however, they may be obtained as perfectly sharp lines if the pressure of the luminous vapor is greatly reduced. In collaboration with Dr. Eberhard I have accordingly made with Spectrograph III a number of photographs of the spectrum of Geissler tubes containing silicon tetrafluoride under low pressure. Basing the determinations upon Kayser's wave-length of the three iron lines at $\lambda 4118.709$, 4132.217,[3] and 4144.033, I obtained the following wave-length of the silicon lines:

Plate III: 528, 530	4128.205	4131.038
531	.201	.036
532	.207	.042
533	.203	.042
Mean	4128.204	4131.040

[1] *Physikalische Zeitschrift*, **4**, 427, 1903.

[2] In the original the wave-length is given by mistake as 4126.5.

[3] Communicated by Professor Kayser in a letter.

The line at 4267 appears very strong and diffuse in the spark spectrum of carbon. The most important earlier determinations of its wave-length are as follows:

Angström and Thalén (*Nova Act. Soc. Upsala*, 1875) - - -	4266.0
Hartley and Adeney (*Phil. Trans.*, 1884)- - - - - -	4266.3
Eder and Valenta (*Denkschriften d. Wiener Akad.*, 1893) - -	4267.5
Deslandres (*Comptes rendus*, 1895) - - - - - - -	4267.0
Exner and Haschek (*Sitzungsberichte d. Wiener Akad.*, 1897) -	4267.10

This wave-length was accordingly hitherto quite inaccurate.

The line may be obtained sharp, if powerful spark discharges (with condenser and spark-gap) are passed through a Geissler tube containing a carbon compound at low pressure. In this way, using a cyanogen tube, I made the following two determinations of the wave-length of this line some time ago.

Plate III:	495 - - - - - -	4267.304
	496 - - - - -	.298
	Mean - - -	4267.301

This value is also referred to Kayser's standards.

Even in a vacuum the line always remains rather broad, and it gives the impression of duplicity, the components not being resolvable with the dispersion employed. The determination of the wave-length is therefore somewhat uncertain.

POTSDAM,
April 13, 1903.

SOME MISCELLANEOUS RADIAL VELOCITY DETERMINATIONS WITH THE BRUCE SPECTROGRAPH.

By WALTER S. ADAMS.

THE RADIAL VELOCITIES OF THE TWO COMPONENTS OF 61 *CYGNI.*

THE recent adaptation of the Bruce spectrograph for use with one prism has made possible a determination of the radial velocities of the two stars composing 61 *Cygni* which is of interest because of its important bearing upon the question of the physical connection and relative parallax of this celebrated pair. The spectrograms obtained are as follows:

61^1 *CYGNI.*

Plate	Date	No. of Lines	Velocity
B 387	1902, August 11	17	−63 km
DB^1 25	1903, May 17	13	61
DB^1 36	June 6	17	63
		Mean	−62 km

62^2 *CYGNI.*

Plate	Date	No. of Lines	Velocity
C 13	1903, January 9	9	−63 km
DB^1 10	May 7	13	65
DB^1 32	June 5	16	61
		Mean	−63 km

Plate B 387 was taken with the regular three-prism train and the 24-inch camera; plate C 13, with the same prism train and a 10.5-inch camera; the remainder of the plates, with a single prism and the 24-inch camera.

The degree of accuracy attained is probably somewhat higher in the case of 61^1. The error of the mean should, however, in neither case exceed about 3 km. A value of − 54 km was obtained for 61^1 *Cygni* by Bélopolsky at Pulkowa in 1895, based

upon the measurement of five lines on each of two plates. This constitutes the only previously published determination for either of the stars.

The agreement of the radial velocities of the two components, when considered in connection with their similar proper motions, would seem to indicate that the stars are unquestionably physically connected. Their real motion in reference to the Sun, under the assumption of a parallax of 0".4 and a proper motion of 5".2, would be about 80 km, or in space, when corrected for the solar motion, about 64 km.

THE RADIAL VELOCITY OF ϵ *URSAE MAJORIS.*

The bright star ϵ *Ursae Majoris* was included in the list of stars whose radial velocities were determined by Vogel and Scheiner at Potsdam in 1889–90. The mean of their values is −30.4 km for the epoch 1889.39. A plate obtained with the Bruce spectrograph in April 1902 gave a value of −8 km. While the spectrum of this star is of the Ia2 type of Vogel's classification with faint and broad metallic lines whose accurate measurement is difficult, a discrepancy of this amount is too large to be accounted for in such a way, and is not encountered among other stars of the Potsdam list having similar spectra. Accordingly the star was placed upon the observing list, and the following plates were obtained:

Plate	Date	No. of Lines	Velocity
B 339	1902, April 30	10	−8 km
B 344	May 14	8	8
B 364	June 20	9	8
A 357	July 23	7	8
B 479	Dec. 31	12	11
B 496	1903, March 24	16	10
A 422	April 2	13	10
A 431	April 8	13	10
A 440	April 16	12	11
		Mean	−9.4 km

The accordance of these measures is entirely satisfactory for a star with this type of spectrum, and it would appear that no appreciable change has taken place in the star's radial velocity

during the interval of a year covered by the observations. The difference as compared with the Potsdam results consequently is not accounted for, but it seems quite possible that we may here be dealing with a spectroscopic binary of much longer period than any met with hitherto.

THE VARIABLE VELOCITY OF β *SCORPII* IN THE LINE OF SIGHT.

Four spectrograms give the following values of the radial velocity of this bright star:

Plate	Date	No. of Lines	Velocity
B 327	1902, April 16	7	− 9 km
A 439	1903, April 16	4	+19
DB[1] 29	June 5	5	−99
DB[1] 42	June 12	4	−97

The spectrum is of the *Orion* type, but all the lines are exceedingly broad, and the measures are uncertain to the extent of several kilometers. The use of low dispersion in photographing the spectrum of this star has proved of decided advantage, the gain in narrowness of the lines more than counteracting the effect of the reduced scale.

THE VARIABLE VELOCITY OF ϵ *HERCULIS* IN THE LINE OF SIGHT.

The spectrum of this star appears to be composite, and its variations will be made the subject of further investigation. Three spectrograms furnish the following velocity for the star which gives the stronger lines, the spectrum being of the Ia2 type:

Plate	Date	No. of Lines	Velocity
A 449	1903, April 30	4	−58 km
DB[1] 17	May 7	4	−43
DB[1] 33	June 6	8	−22

These measures are to be regarded as preliminary and may be changed considerably in a further discussion of the star's motion.

YERKES OBSERVATORY,
June 15, 1903.

Minor Contributions and Notes.

A PHOTOGRAPHIC MAP OF THE ENTIRE SKY.[1]

THE collection of photographs at the Harvard College Observatory contains, in addition to the plates taken with the larger instruments, numerous photographs taken with two small anastigmatic lenses, each having an aperture of one inch, and a focal length of about thirteen inches. A region of more than thirty degrees square is covered by a single eight by ten inch plate. With exposures of one hour, stars as faint as the twelfth magnitude are, in some cases, obtained. Owing to the scale of the plates, identification of the individual stars would become difficult if, by using longer exposures, the number of stars were increased. One of these lenses is mounted at Cambridge, and is used principally for the northern stars. The other is similarly used for the southern stars at Arequipa. At each station two sets of photographs have been taken, the first having centers in declinations 0°, 30°, 60°, and 90°, and the second in declinations 15°, 45°, and 75°, the centers of the second set coinciding as nearly as possible with the corners of the first. An attempt is made to cover all parts of the sky, not too near the Sun, at least twice each month, once with each set. These photographs have proved unexpectedly useful here for determining the past as well as the present changes in light of variable stars, new stars, and similar objects. (See *Circular* No. 69, and elsewhere.) Of course, the small scale diminishes their value for measuring positions, although the minuteness of the images in part compensates for this difficulty.

The amount of useful material contained in these plates is so great that we are able to extract but a small portion of it, although an appropriation from the Carnegie Institution has, this year, permitted a great increase to be made. Various plans have been considered for placing copies of the photographs thus collected within the reach of astronomers. It was at first proposed to print a series of engravings on the same scale as the charts of the *Durchmusterung*. Numerous difficulties presented themselves, especially if an attempt was made to engrave the parallels and meridians upon the charts. The defects

[1] *Harvard College Observatory Circular* No. 71.

introduced by paper and ink are very troublesome, and are likely to differ in different copies. The variation in intensity of images which can be represented on paper is small compared with that on glass, and, finally, the expense would be large. The advantages of glass negatives are very great, especially to one accustomed to use them. They can be reproduced by contact printing so as to give results but slightly inferior to the original. A single contact print, forming a positive, with bright stars on a dark background, although more nearly resembling the sky itself, does not prove convenient in actual use. It cannot be superposed upon another positive, nor upon a paper map. A double contact print, however, furnishes a negative which is for some purposes nearly as useful as the original. Measurements of position or intensity of the images can be made, the cost is not large, and is nearly proportional to the number of copies furnished. A set of fifty-five of these prints on glass, covering the sky from the north to the south pole, has accordingly been prepared, and is described in Table I. A current number for designating the plate, and the approximate right ascension and declination of the center, are given in the first three columns. The designation of the original negative is given in the fourth column. It consists of the letters indicating the series, and the number of the plate in that series. The plates taken with the anastigmatic lens at Cambridge are indicated by the letters AC, those at Arequipa by AM. The date, the Greenwich Mean Time, and the length of exposure, expressed in minutes, are given in the next three columns. Remarks on some of the plates follow the table. The position of any particular object is indicated by two numbers placed in brackets. The first gives the distance of the object, in millimeters, from the left-hand edge of the exposed portion of the plate. The second number gives the corresponding distance from the lower edge of the exposed portion.

In the use of these plates certain suggestions may be made, especially for the benefit of those unaccustomed to astronomical photographs. To compare them with the sky, they should be examined with the glass side toward the eye. Objects of interest may then be conveniently marked on this side of the plate with a pen. The label is on the southern end of each plate, excepting in the cases of Nos. 1 and 54, which contain the north and south poles, respectively. Conspicuous configurations, like those of *Orion*, *Ursa Major*, and *Scorpius*, are easily recognized by inspection, aided when necessary by a small atlas. For the faint stars, a comparison may be made with

TABLE I.

CATALOGUE OF PLATES.

No.	R. A.	Dec.	Negative	Date	G. M. T.	Ex.
	h. m.	°		y. m. d.	h. m.	m.
1	0 00	+90	AC 2161	1902, January 4	22 43	39
2	0 00	+60	AC 1943	1901, November 2	17 51	63
3	3 00	+60	....	....	.. ..	..
4	6 00	+60	....	....	.. ..	..
5	9 00	+60	AC 3625	1903, May 13	13 16	59
6	12 00	+60	AC 3466	1903, March 31	18 42	71
7	15 00	+60	AC 3620	1903, May 12	14 51	58
8	18 00	+60	AC 3629	1903, May 13	18 14	70
9	21 00	+60	AC 3630	1903, May 13	19 25	60
10	0 00	+30	AC 2152	1902, January 4	13 06	57
11	2 00	+30	AC 124	1898, December 16	13 09	62
12	4 00	+30	AC 2155	1902, January 4	16 16	59
13	6 00	+30	AC 3469	1903, April 1	12 48	75
14	8 00	+30	AC 3363	1903, February 26	16 16	59
15	10 00	+30	AC 2160	1902, January 4	21 55	56
16	12 00	+30	AC 3332	1903, February 20	21 04	67
17	14 00	+30	AC 1105	1900, December 28	22 04	68
18	16 00	+30	AC 252	1899, March 20	20 19	64
19	18 00	+30	AC 1013	1900, September 24	12 12	56
20	20 00	+30	AC 3353	1903, February 23	22 03	62
21	22 00	+30	AC 1014	1900, September 24	13 16	69
22	0 00	0	AM 1569	1902, September 25	15 32	58
23	2 00	0	AM 151	1899, September 14	18 45	60
24	4 00	0	AC 1093	1900, December 27	13 27	71
25	6 00	0	AC 2156	1902, January 4	17 22	73
26	8 00	0	AC 513	1899, November 9	22 01	64
27	10 00	0	AC 2518	1902, May 2	14 19	65
28	12 00	0	AM 1431	1902, July 8	12 09	65
29	14 00	0	AC 2547	1902, May 13	16 35	73
30	16 00	0	AM 1420	1902, July 3	13 15	60
31	18 00	0	AM 1436	1902, July 9	14 18	60
32	20 00	0	AM 1439	1902, July 9	17 30	60
33	22 00	0	AM 1441	1902, July 9	19 39	60
34	0 00	−30	AM 1427	1902, July 3	20 53	60
35	2 00	−30	AM 1451	1902, July 10	20 56	61
36	4 00	−30	AM 1802	1903, January 16	15 55	68
37	6 00	−30	AM 1785	1903, January 3	17 33	60
38	8 00	−30	AM 1798	1903, January 15	16 35	60
39	10 00	−30	AM 1419	1902, July 3	12 01	61
40	12 00	−30	AM 1443	1902, July 10	12 07	67
41	14 00	−30	AM 1444	1902, July 10	13 15	60
42	16 00	−30	AM 1452	1902, July 11	14 39	60
43	18 00	−30	AM 1461	1902, July 12	14 37	60
44	20 00	−30	AM 1463	1902, July 12	17 26	61
45	22 00	−30	AM 1440	1902, July 9	18 36	61
46	0 00	−60	AM 703	1900, November 1	13 37	61
47	3 00	−60	AM 1782	1903, January 3	14 12	64
48	6 00	−60	AM 712	1900, November 10	18 33	60
49	9 00	−60	AM 1247	1902, May 15	12 43	60
50	12 00	−60	AM 459	1900, May 8	15 10	72
51	15 00	−60	AM 1469	1902, July 14	14 13	60
52	18 00	−60	AM 809	1901, May 27	17 48	61
53	21 00	−60	AM 1464	1902, July 12	18 32	60
54	0 00	−85	AM 626	1900, September 3	17 11	60
55	14 00	−75	AM 1389	1902, June 25	15 13	60

the charts of the *Durchmusterung*, which are on nearly four times the scale. The plates may be examined with a reading glass or a two-inch positive eyepiece. The latter will, in almost all cases, serve to distinguish defects from actual star images. It will be noticed that images of bright stars are surrounded by sixteen rays, due to the iris diaphragm of the lens.

REMARKS.

1. The position of the North Pole is [95, 104]. [97, 111], the Pole Star. The circular form and absence of diffraction rays show that [110, 118] is a defect.

2. See No. 10.

3. Owing to a change in the scheme of work, photographs have not been obtained in the position of this and of the following region. These omissions will be supplied in a few weeks, or as soon as their positions permit photographs to be obtained.

4. See No. 3.

6. [25, 89], ζ *Ursae Majoris;* also on No. 7 [172, 85].

7. See No. 6.

10. [48, 190], Nebula in *Andromeda;* also on No. 2 [47, 11].

12. [117, 69], *Pleiades*, showing also the nebulosity surrounding them. Star l and perhaps m of the sequence given in *Harvard Annals*, XVIII, p. 153, appears on this plate. [131, 185], *Nova Persei*, No. 2. Nos. 54, 58, and 64 of Hagen's Catalogue are readily seen.

13. [80, 58, *Neptune*. [39, 107], *Nova Geminorum*. Stars t and w of *Circular* No. 70 (*Hagen* 73 and 77) appear on this plate. Numerous clusters are also shown where it is crossed by the Milky Way. [82, 71], the cluster *N. G. C.* 2129.

14. [40, 48], *Praesepe.*

17. [115, 92], the cluster *Messier* 3, N. G. C. 5272.

18. [51, 125], *Messier* 13, *N. G. C.* 6205, the cluster in *Hercules.*

19. [46, 163], the double stars ε and 5, *Lyrae;* also on Plate 20 [180, 169]. [31, 124]. *Messier* 57, *N. G. C.* 6720. The Ring Nebula in *Lyra;* also on Plate 20 [175, 127].

20. See No. 19.

25. [134, 80], *N. G. C.* 1976, the great Nebula in *Orion*, [126, 91], ζ *Orionis* showing the faint nebulosity, *N. G. C.* 2024, north following it. [106, 105], α *Orionis* The red color of this star is well shown by its faintness on the photograph. [40, 161], the cluster *Messier* 14, *N. G. C.* 2287. [31, 3], *Sirius.* The circle around this, and other very bright stars, is due to light reflected from the back of the plate.

30. [167, 116], the cluster *Messier* 5, *N. G. C.* 5904.

37. [74, 83], *Pallas.*

40. The fogging on the northern edge is due to the Moon.

42. [68, 119], α *Scorpii.* The red color of this star is well shown by its faintness on the photograph.

43. [166, 141], *Uranus.* [126, 89], the cluster *Messier* 7, *N. G. C.* 6475. [109, 75], the cluster *Messier* 6, *N. G. C.* 6405. [99, 144], the Trifid Nebula, N. G. C. 6514. See *Annals* XXVI, p. 204, and Plate III. [96, 136], *N. G. C.* 6523.

44. [127, 136], *Vesta.* [113, 149], *Saturn.*

45. [169, 169], *Jupiter*. Satellite III is seen about half a millimeter to the left of it

46. [87, 25], the cluster 47 *Tucanae, N. G. C.* 104. [76, 17], the Small Magellanic Cloud. See *Annals* XXVI, p. 205, and Plate IV. [24, 102], *Achernar* α *Eridani*. Next to *Sirius*, this is the brightest star in the sky; also on No. 47 [167, 102].

48. [115, 40], the Large Magellanic Cloud. See *Annals*, XXVI, p. 206, and Plate IV. [77, 137], *Canopus*, α *Carinae*. [13, 73], the cluster *N. G. C.* 2516; also on No. 49 [145, 86].

49. Several fine clusters appear on this plate, which are better shown on Plate 50.

50. [182, 49], the cluster *N. G. C.* 3114; also on Plate 49 [56, 94]. [153, 64], η *Carinae* or η *Argus*; also on Plate 49 [24, 88]. This is one of the most remarkable regions in the sky. See *Annals* XXVI, p. 206, and Plates V and VII. [138, 74]. *N. G. C.* 3523; also on Plate 49, [6, 86]. [113, 61], *N. G. C.* 3766. The Southern Cross is well shown on this plate. See also *Annals* XXVI, p. 203, and Plate II. [85, 79], δ *Crucis*. [79, 52], α *Crucis*. [72, 88], γ *Crucis*. [62, 71], β *Crucis*. The faintness of γ *Crucis* is due to its red color. This renders the Cross less conspicuous on the photograph than to the eye. [58, 66], the cluster κ *Crucis*, *N. G. C.* 4755. [7, 50], β *Centauri*. [6, 134], the cluster ω *Centauri*, *N. G. C.* 5139, the finest globular cluster in the sky.

51. [144, 90], β *Centauri*; also on No. 55 [99, 193]. [118, 91], α *Centauri*; also on No. 55 [72, 188]. So far as known this is the nearest star in the sky. [42, 124], the cluster *N. G. C.* 6067.

54. As the latitude of the Arequipa station is $-16° 22'$, a good photograph could not be obtained if the declination of the center of the plate was at $-90°$, Accordingly, this plate has been taken in R. A. $0^h. 0^m$, December $-8°.36$. Plate 55, taken from the other set of photographs, has been added, to cover the region which would otherwise be omitted. The position of the South Pole is [92, 61]; also on No. 55 [96, 15]. [96, 64], σ *Octantis*; also on No. 55 [91, 14]. [84, 166], 47 *Tucanae*. [73, 157], the Small Magellanic Cloud.

55. See Nos. 51 and 54.

The cost of these plates will prevent the wide, gratuitous distribution of them that is made of the *Annals*. In any case, this would not be advisable, since many would thus be placed where little or no use would be made of them. Copies of this set of photographs, consisting of fifty-five glass negatives, each eight by ten inches, will be supplied for $15; selected sets of ten plates, $3. This is less than the actual cost, the balance being paid from the Advancement of Astronomical Science Fund of the Harvard Observatory. The privilege of increasing the price later is reserved. If the demand justifies it, copies of the second set of plates, whose centers are near the corners of these, will be issued later.

EDWARD C. PICKERING.

MAY 19, 1903.

Reviews

The Theory of Optics. By Paul Drude. Translated from the German by C. Riborg Mann and Robert A. Millikan, of the University of Chicago. New York: Longmans, Green & Co., 1902.

It is a satisfaction to note that there has appeared a translation of this work, which received such instant recognition at the hands of physicists the world over upon its appearance in Germany. The translators deserve much praise for the conscientious and accurate manner in which they have accomplished their task.

As to the subject-matter: Professor Drude has produced a work of the greatest value—distinctly modern and up to date, logical and concise, yet clear. The keynote of the work is sounded by the author in his preface: "My purpose is attained if these pages strengthen the reader in the view that optics is not an old and worn-out branch of physics, but that in it also there pulses a new life whose further nourishing must be inviting to everyone." (The English here is not a fair sample of the work of the translators.)

In detail we may note the following:

The mathematical treatment of the formation of images is very thorough. A brief practical survey of optical instruments (telescope, microscope, photographic lens systems, etc.) is given. The subject of photography in natural colors is merely touched upon.

One of the most valuable and remarkable parts of the book is that which deals with diffraction problems. Starting with Huyghen's elementary light-wave principle, Fresnel's modification of it, and Kirchhoff's further development (simplified by Voigt), the reader is led logically through an exhaustive mathematical discussion of important diffraction phenomena. This is accompanied by a brief treatment of grating, prism, and echelon.

Under polarization phenomena Wiener's stationary light-wave experiment is mentioned and emphasized, and rightly so, for from it the important conclusion follows inevitably that the direction of the light vector is perpendicular to the plane of polarization.

Attention is called to the first brilliant success of the electromagnetic theory of light, that "the velocity of light in ether is equal to the ratio of the electromagnetic to the electrostatic units."

In discussing the subject of dispersion in the magnetic rotation of the plane of polarization, the bearing of the phenomenon known as the

Hall effect is shown, and the Ampère-Weber theory of molecular currents is modified so as to give the correct results. From this discussion we are then led directly to Voigt's theory of the Zeeman effect. The statement is made that the physical significance of Voigt's explanation of anomalous Zeeman effects has not yet been shown.

The "ion hypothesis leads to some new dispersion formulæ for the natural and magnetic rotation of the plane of polarization."

A comprehensive treatment of light problems with reference to bodies in motion leads to the aberration of light and to Michelson's experiment with the refractometer oriented in two positions with respect to the rotation of the earth. This latter is followed by the rather startling explanatory proposition advanced by Lorentz and by Fitzgerald, that the length of a solid body may depend upon its absolute motion in space.

Part III, "Radiation," covering in the original German but fifty-seven pages, and in the translation sixty, is a fine piece of work. The subject treated might well demand an entirely separate book; yet Professor Drude's discussion is clear and logical.

The electromagnetic theory is adopted as fulfilling most completely the test of experiment; and as Professor Michelson says in his preface to the English edition, "No complete development of the electromagnetic theory in all its bearings, and no comprehensive discussion of the relation between the laws of radiation and the principles of theorenodynamics have yet been attempted in any general text in English."

Throughout the whole treatise we discern a strong tendency to view nature as a mechanism. The discussion and interpretation of equations resulting from mathematical reasoning are both able and clear. Descriptively, the book is fully on a par with Preston's *Theory of Light* and mathematically more valuable, as well as more lucid and attractive, than Basset's *Treatise on Physical Optics.*

The general criticism may be made that the book is unbalanced, too little space being given to some subjects, too much to others. This, however, is quite a pardonable fault, especially as the author makes "no claim to such completeness as is aimed at in Mascart's excellent treatise, or in Winkelmann's *Handbuch*," and writes: "For the sake of brevity I have passed over many interesting and important fields of optical investigation."

Professor Drude's *Theory of Optics* should be assigned a most prominent place in the library of every modern physicist—a place which it should hold for many a year despite the rapid progress of the science.

N. A. K.